MATH243 January 2002 EXAMINATION ANSWERS

1. CRE: $u_x = v_y$, $u_y = -v_x$.
$u = 4x^4$, $v = 0$.
$f = 3xy + i(-\frac{3}{2}x^2 + \frac{3}{2}y^2 + C) = -\frac{3}{2}iz^2 + iC$, $C \in \mathbf{R}$.

2. $-4i - 6$.

3. $\pi/6$ by CRT; 0 by CRT; 0 by CT.

4. (i) $1 - \frac{1}{2}z^2 + \frac{5}{24}z^4$ (ii) $1 - z^2 + \frac{11}{6}z^4$

5. (a) double pole; (b) removable, $\frac{16}{\pi^3}$; (c) simple pole, $-\frac{16}{\pi^2}$.

6. (a) $Res_{-3i/4} = -i/7$, $Res_{-4i/3} = i/7$; (b) $2\pi/7$.

7. (a) $u_{xx} + u_{yy} = 0$; (b) $k = 1$, $v = e^y \cos x + x$; (c) $g = f' = v_y + iv_x = e^{-iz} + i$.

8. (i) $\frac{z^2}{z^2-1}$, $\frac{z}{(z^2-1)^2}$; (ii) $R = 1/3$; converges everywhere on $|z| = 1/3$.

9. (a) poles $z = 6, -5$; (b) $\sum_{n\geq 0} \frac{4^n}{(z-2)^{n+1}} + \sum_{k\geq 0} \frac{(z-2)^k}{(-7)^{k+1}}$; (c) diverges.

10. $\frac{\pi}{2e^a}$.

11. $\frac{\pi}{36}(\cos 15 - e^{-15})$.