PAPER CODE NO.
MATH199

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THE UNIVERSITY
of LIVERPOOL

AUGUST/SEPTEMBER 2007 EXAMINATIONS

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Bachelor of Engineering   :    Year 1
  Bachelor of Science   :    Year 2
 Master of Engineering   :    Year 1
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MATHEMATICAL TECHNIQUES FOR ENGINEERS

TIME ALLOWED : Three Hours

INSTRUCTIONS TO CANDIDATES

You may attempt all questions. All answers to Section A and to the best THREE questions from Section B will be taken into account. Section A carries 55% of the available marks.

Your attention is drawn to the formula sheet which accompanies this exam paper.

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SECTION A

1. Differentiate the following functions with respect to x, simplifying your answers as much as possible:

$$\text{(i)}\, x^2, \qquad \text{(ii)}\, x \ln x + x^4 \sin(2x), \qquad \text{(iii)}\, \frac{x+1}{(x+2)^2}.$$

[6 marks]

2. Sketch the graph of $y = (x-4)/(2x+6)$. Mark the coordinates of any points where the graph crosses the axes, any asymptotes and any stationary points. [6 marks]

3. Evaluate:

$$\text{(i)} \int (x^2 - 2x)dx\,, \qquad \text{(ii)} \int \frac{3}{x^2+25}dx\,, \qquad \text{(iii)} \int_4^5 \frac{2}{x^2-4}dx\,.$$

[6 marks]

4. Use the substitution $u = x^2 + 4$ to evaluate the indefinite integral

$$\int \frac{x}{(x^2+4)}dx\,.$$

[3 marks]

5. Solve the differential equation

$$\frac{dy}{dt} = 5y$$

given that $y = 3$ when $t = 0$. [3 marks]

6. Find the value of the number λ such that the vector $3\mathbf{i} + \mathbf{j} - 4\mathbf{k}$ is orthogonal to $\mathbf{i} - \lambda\mathbf{j} + 2\mathbf{k}$. [3 marks]

7. Given the vectors $\mathbf{a} = \mathbf{i} - \mathbf{j}$ and $\mathbf{b} = 2\mathbf{j} + 3\mathbf{k}$,

   (i) find the vector $\mathbf{a} - \mathbf{b}$ and determine its magnitude to 2 decimal places;

   (ii) find the angle, to the nearest degree, between $\mathbf{a} - \mathbf{b}$ and $\mathbf{b}$;

   (iii) evaluate $\mathbf{a} \times \mathbf{b}$.

[7 marks]

8. The vertices O, A, B of a triangle have coordinates $(0, 0, 0)$, $(1, -2, 1)$ and $(2, 0, -3)$ respectively. Calculate the area of triangle OAB to 2 decimal places.

[5 marks]

9. Write $z = 3 + 4i$ and $w = 1 + i$ in the form $re^{i\theta}$. Calculate in the same form z/w. Find the angle between the two complex numbers z and w in the Argand plane.

[6 marks]

10. Sketch the level curves $w = 0, 1$ and 2 of the function $w = y^2 - x^2 + 1$.

[4 marks]

11. Find all first order and second order partial derivatives with respect to x and y of the function

$$w = \sin(x + y) + \cos(x - y)\,,$$

and verify that

$$\frac{\partial^2 w}{\partial x \partial y} = \frac{\partial^2 w}{\partial y \partial x}\,.$$

[6 marks]

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SECTION B

12. Consider the function

$$y = \frac{2e^x}{x-1}.$$

Does this function have any maxima or minima? If so where are they?

[5 marks]

Sketch the graph of y. Include on your sketch the coordinates of any points where the curve crosses the axes, the coordinates of any stationary points and the equations of any asymptotes. [10 marks]

13. (i) Solve the differential equation

$$\frac{dy}{dx} = 3y + 5\sin(2x)\,.$$

[6 marks]

(ii) Sketch the solid formed by rotating the curve $y = e^{-x}$ around the x-axis by 2π, between $x = 0$ and $x = 2$. Calculate the volume of this solid. [9 marks]

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14. The coordinates of the points A, B and C are $(5,-1,2)$, $(3,1,1)$ and $(3,2,-2)$ respectively.

    (i) Write down the line vectors $\overrightarrow{AB}$ and $\overrightarrow{BC}$. [3 marks]

    (ii) Find the vector form of the equation of the straight line which passes through the points B and C. [3 marks]

    (iii) Find a vector that is perpendicular to the line BC and the equation of a straight line passing through C that is perpendicular to the line BC. [6 marks]

    (iv) Determine whether the point $(2,3,1)$ lies on the line BC, justifying your answer. [3 marks]

15. (i) Sketch the region $|z-3i| = 2$ in the complex plane where $z = x+iy$. Where does this region cross the imaginary axis? [4 marks]

    (ii) Use de Moivre's theorem,

$$(\cos\theta + i\sin\theta)^n = (\cos n\theta + i\sin n\theta),$$

    to write $\sin(3\theta)$ in terms of $\sin^3\theta$ and $\sin\theta$. [6 marks]

    (iii) Given the harmonic function

$$V(t) = 4\cos(\frac{\pi t}{3} + \pi/4)$$

    write down $V(t)$ in the form $V(t) = A\cos(\frac{\pi t}{3}) + B\sin(\frac{\pi t}{3})$. For what values of t does $V(t) = 0$? [5 marks]

16. Sketch the level curves $w = 1$, $w = 2$ and $w = 4$ of the function $w = x^2 + 4y^2$. [5 marks]

    (i) Find the rate of change of w at the point $(2,1)$, in the outward radial direction. [5 marks]

    (ii) The level curve $w = 3$ has the point $(1, 1/\sqrt{2})$ lying on it. Find the equation of the straight line that is perpendicular to the tangent at this point.

    [5 marks]