

PAPER CODE NO.
MATH199



THE UNIVERSITY
of LIVERPOOL

SUMMER 2005 EXAMINATIONS

Bachelor of Engineering : Year 1
Bachelor of Science : Year 1
Bachelor of Science : Year 2
Master of Engineering : Year 1

MATHEMATICAL TECHNIQUES FOR ENGINEERS

TIME ALLOWED : Three Hours

INSTRUCTIONS TO CANDIDATES

You may attempt all questions. All answers to Section A and to the best THREE questions from Section B will be taken into account. Section A carries 55% of the available marks.

Your attention is drawn to the formula list which accompanies this exam paper.



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SECTION A

1. Given $\mathbf{a} = 2\mathbf{i} - 7\mathbf{j} + 3\mathbf{k}$ and $\mathbf{b} = -\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}$ find $\mathbf{a} - 3\mathbf{b}$. Hence, determine to three decimal places, the magnitude of $\mathbf{a} - 3\mathbf{b}$ and to the nearest degree, the angle between $\mathbf{a} - 3\mathbf{b}$ and the positive y -axis.

[4 marks]

2. Find the value of n for which the vectors $2\mathbf{i} + 3\mathbf{j} + n\mathbf{k}$ and $-\mathbf{i} + 5\mathbf{j} - 6\mathbf{k}$ are orthogonal.

[3 marks]

3. Differentiate the following with respect to x , simplifying your answer,

$$(i) \quad 4x^7 e^{-3x} \quad , \quad (ii) \quad \frac{x^2}{(4x + 5)^2} .$$

[5 marks]

4. Sketch the graph of $y = e^{3x} + 3$. Include in your graph the coordinates of the points, if any, where the graph crosses the x - and y - axes and the equation of the asymptote.

[4 marks]

5. Sketch the level curves $w = -1$, 0 and 1 of $w = y - 3x^2$.

[4 marks]

6. Given

$$w = \cos(x^3 - y)$$

find

$$\frac{\partial w}{\partial x} \quad , \quad \frac{\partial w}{\partial y} \quad , \quad \frac{\partial^2 w}{\partial x^2} \quad , \quad \frac{\partial^2 w}{\partial y^2} \quad \text{and} \quad \frac{\partial^2 w}{\partial x \partial y} .$$

[5 marks]

7. The number of defects, N , in a material of volume V at temperature T is $N = qT^4/V^2$ where q is a constant. The volume fluctuates by $\pm 0.15\%$ and the temperature (independently) by $\pm 0.12\%$. Find the approximate resultant percentage fluctuation of N .

[4 marks]



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8. Express $(2 + 3i)/(3 + i)$ in the form $a + ib$ where a and b are real numbers.
[3 marks]

9. Given that $w = z^2 - iz$ where $w = u + iv$ and $z = x + iy$, and u, v, x and y are real, find u and v as functions of x and y .
[3 marks]

10. Determine

(i) $\int \left(4x^3 - \frac{7}{x} + 5\right) dx$
[3 marks]

(ii) $\int \frac{3}{(4x^2 + 49)} dx$
[3 marks]

(iii) $\int 4xe^{-5x} dx$
[4 marks]

(iv) $\int 4x^2 \cos(x^3 - 2) dx$ (change variable to $u = x^3 - 2$)
[4 marks]

(v) $\int_0^{\infty} e^{-x} \sin x dx$.
[3 marks]

11. Solve the differential equation

$$\frac{dy}{dx} = \frac{1}{(4x + 1)}$$

given that $y = 3$ when $x = 0$.

[3 marks]



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SECTION B

12. The coordinates of the points A , B and C are $(3, 4, 5)$, $(1, -2, 1)$ and $(7, 2, 3)$ respectively.

(i) Write down the line vectors \overrightarrow{AB} and \overrightarrow{BC} .

[3 marks]

(ii) Hence, find to the nearest degree the angle between the lines AB and BC .

[4 marks]

(iii) Find the vector form and the Cartesian form of the equation of the straight line which passes through the points B and C .

[4 marks]

(iv) Determine the point where the line $\mathbf{r} = (-2, 5, 4) + \mu(0, -18, -8)$ intersects the line through B and C where μ is a parameter.

[4 marks]

13. Given

$$y = \frac{2x^2}{(x^2 - 16)}$$

show that

$$\frac{dy}{dx} = -\frac{64x}{(x^2 - 16)^2}.$$

[4 marks]

Sketch the graph of y . Include on your graph the coordinates of all stationary points, the coordinates of any points where the graph crosses the axes and the equations of all the asymptotes.

[11 marks]



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14. Given that $w = -x^2 + 2y^2 + 6xy + 4y + 3$ find

$$\frac{\partial w}{\partial x}, \quad \frac{\partial w}{\partial y}, \quad \frac{\partial^2 w}{\partial x^2}, \quad \frac{\partial^2 w}{\partial y^2} \quad \text{and} \quad \frac{\partial^2 w}{\partial x \partial y}.$$

[5 marks]

Find and classify any stationary points of w .

[6 marks]

Find the rate of change of w at the point $(3, 4)$ in the outward radial direction.

[4 marks]

15. (i) Working to three decimal places, find a polar and an exponential form for the complex numbers $(2 - i)$ and $(1 + 4i)$. Hence, working to three decimal places find a polar and exponential form for

$$\frac{(2 - i)^2}{(1 + 4i)^3}.$$

[11 marks]

(ii) Given the harmonic function

$$V(t) = 2 \cos(4t) - 7 \sin(4t)$$

write down its amplitude and frequency.

Express $V(t)$ as a cosine harmonic function, evaluating its phase angle to three decimal places.

[4 marks]

16. (i) Solve the differential equation

$$\frac{dy}{dx} = 3y - e^{-3x} \sin(4x).$$

[9 marks]

(ii) Solve the differential equation

$$\frac{d^2 y}{dx^2} = -81y$$

given that $y = 4$ and $dy/dx = -2$ when $x = 0$.

[6 marks]