PAPER CODE NO.
MATH199

THE UNIVERSITY
of LIVERPOOL

SUMMER 2004 EXAMINATIONS

Bachelor of Engineering : Year 1
Bachelor of Science : Year 1
Bachelor of Science : Year 2
Master of Engineering : Year 1

MATHEMATICAL TECHNIQUES FOR ENGINEERS

TIME ALLOWED : Three Hours

INSTRUCTIONS TO CANDIDATES

You may attempt all questions. All answers to Section A and to the best THREE questions from Section B will be taken into account. Section A carries 55% of the available marks.
Your attention is drawn to the formula list which accompanies this exam paper.

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SECTION A

1. Given $\mathbf{a} = 4\mathbf{i} + 5\mathbf{j} - 2\mathbf{k}$ and $\mathbf{b} = 3\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ find $2\mathbf{a} - \mathbf{b}$. Hence, determine to three decimal places, the magnitude of $2\mathbf{a} - \mathbf{b}$ and to the nearest degree, the angle between $2\mathbf{a} - \mathbf{b}$ and the z-axis.

[**4 marks**]

2. Find the scalar product of the vectors $3\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$ and $2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$. Find to the nearest degree the angle between these two vectors.

[**3 marks**]

3. Differentiate the following with respect to x, simplifying your answer,

$$\text{(i)} \quad \sin(x^3) \quad , \quad \text{(ii)} \quad \frac{2x^2}{(x-3)^3} \ .$$

[**5 marks**]

4. Sketch the graph of $y = e^{-x} - 2$. Include in your graph the coordinates of the points where the graph crosses the x- and y- axes and the equation of the asymptote.

[**4 marks**]

5. Given

$$w = 2x^2 - 4xy + y^2 + 4x - 3y + 7$$

find

$$\frac{\partial w}{\partial x} \ , \ \frac{\partial w}{\partial y} \ , \ \frac{\partial^2 w}{\partial x^2} \ , \ \frac{\partial^2 w}{\partial y^2} \ \text{and} \ \frac{\partial^2 w}{\partial x \partial y} \ .$$

Find and classify any stationary points of the function.

[**9 marks**]

6. The number of defects, n, in a material of volume V at temperature T is $n = cT^2/V^3$ where c is a constant. The volume fluctuates by $\pm 0.10\%$ and the temperature (independently) by $\pm 0.20\%$. Find the approximate resultant percentage fluctuation of n.

[**4 marks**]

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7. Express $(1+i)/(1-i)$ in the form $a+ib$ where a and b are real numbers.
[**3 marks**]

8. Express in terms of the complex variable $z = x + iy$ the interior of the circle centred on $(1, 1)$ with radius 2.
[**3 marks**]

9. Determine

(i) $\int \left(x^2 + \frac{2}{x}\right) dx$

[**3 marks**]

(ii) $\int \frac{4}{\sqrt{(x^2+25)}}\, dx$

[**3 marks**]

(iii) $\int x \sin(3x)\, dx$

[**4 marks**]

(iv) $\int x \sin(x^2-1)\, dx$ (change variable to $u = x^2 - 1$)

[**4 marks**]

(v) $\int_0^\infty 2e^{-2x}\, dx$.

[**3 marks**]

10. Solve the differential equation

$$\frac{dy}{dx} = 5y$$

given that $y = 6$ when $x = 0$.
[**3 marks**]

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SECTION B

11. The coordinates of the points A, B and C are $(2, 5, -2)$, $(1, 7, 3)$ and $(-3, 3, 2)$ respectively.

(i) Write down the line vectors $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

[3 marks]

(ii) Hence, find to the nearest degree the angle between the lines AB and BC.

[4 marks]

(iii) Find the vector form, the Cartesian parametric form and the Cartesian form of the equation of the straight line which passes through the points B and C.

[5 marks]

(iv) Does the point $(-5, 1, -3/2)$ lie on the line through B and C?

[3 marks]

12. Given

$$y = \frac{x^2}{(x-4)}$$

show that

$$\frac{dy}{dx} = \frac{x(x-8)}{(x-4)^2} .$$

[3 marks]

Sketch the graph of y. Include on your graph the coordinates of all stationary points, the coordinates of any points where the graph crosses the axes and the equations of all the asymptotes.

[12 marks]

13. Sketch the level curves $w = -1$, 0 and 1 of the function $w = y + 3x^2$.

[**5 marks**]

(i) Find the rate of change of w at the point $(2, 3)$ in the outward radial direction.

[**6 marks**]

(ii) Find the rate of change of w at the point $(4, 5)$ in the direction towards the point $(9, 7)$.

[**4 marks**]

14. Given the harmonic function $V(t) = 3\cos(2t) + 5\sin(2t)$, write down its amplitude and period.

[**2 marks**]

Sketch $V(t)$ as a function of t. Show on the graph the maximum and minimum values of $V(t)$ and the period of $V(t)$.

[**3 marks**]

Express $V(t)$ as a cosine harmonic, evaluating the phase angle to three decimal places.

[**4 marks**]

Hence find the values of t at which $V(t)$ has its maximum value.

[**3 marks**]

Express $V(t)$ in terms of a complex harmonic function. Write down the complex amplitude of this complex harmonic function.

[**3 marks**]

15. (i) Solve the differential equation

$$\frac{dy}{dx} = -4y + e^x \cos(2x) \, .$$

[**9 marks**]

(ii) Solve the differential equation

$$\frac{d^2y}{dx^2} = 9y$$

given that $y = 2$ and $dy/dx = 3$ when $x = 0$.

[**6 marks**]