PAPER CODE NO.
MATH199

THE UNIVERSITY
of LIVERPOOL

SUMMER 2003 EXAMINATIONS

Bachelor of Engineering : Year 1
Bachelor of Science : Year 1
Master of Engineering : Year 1

MATHEMATICAL TECHNIQUES FOR ENGINEERS

TIME ALLOWED : Three Hours

INSTRUCTIONS TO CANDIDATES

You may attempt all questions. All answers to Section A and to the best THREE questions from Section B will be taken into account. Section A carries 55% of the available marks.
Your attention is drawn to the formula list which accompanies this exam paper.

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SECTION A

1. Given $\mathbf{a} = 3\mathbf{i} - 9\mathbf{j} + 2\mathbf{k}$ and $\mathbf{b} = 2\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$ find $2\mathbf{a} + 3\mathbf{b}$. Hence, determine to three decimal places, the magnitude of $2\mathbf{a} + 3\mathbf{b}$ and to the nearest degree, the angle between $2\mathbf{a} + 3\mathbf{b}$ and the y-axis.

[4 marks]

2. Find the value of the number n such that $3\mathbf{i} + n\mathbf{j} - 5\mathbf{k}$ is orthogonal to $-4\mathbf{i} + 7\mathbf{j} - 6\mathbf{k}$.

[3 marks]

3. Differentiate the following with respect to x, simplifying your answer,

$$\text{(i)} \quad 4x^3\cos(3x) \quad , \quad \text{(ii)} \quad \frac{6x}{(x-7)^2} \ .$$

[5 marks]

4. Sketch the graph of $y = (x^2 - 1)^2$. Include in your graph the coordinates of the points where the graph crosses the x- and y- axes and the coordinates of the stationary points.

[5 marks]

5. Sketch the level curves $w = -1$, 0 and 1 of the function $w = (y-3)/x^2$.

[4 marks]

6. Given that $w = 3x^2y^5 - 2x$, find

$$\frac{\partial w}{\partial x} \ , \ \frac{\partial w}{\partial y} \ , \ \frac{\partial^2 w}{\partial x^2} \ \text{and} \ \frac{\partial^2 w}{\partial x \partial y} \ .$$

[4 marks]

7. The number of defects, n, in a material of volume V at temperature T is $n = cT^3/V^2$ where c is a constant. The volume fluctuates by $\pm 0.2\%$ and the temperature (independently) by $\pm 0.3\%$. Find the approximate resultant percentage fluctuation of n.

[4 marks]

8. Express $2/(1+4i)^2$ in the form $a+ib$ where a and b are real numbers.

[**3 marks**]

9. Given that $w = 2z^2 + z - 3$ where $w = u + iv$ and $z = x + iy$ and u, v, x and y are real, find u and v as functions of x and y.

[**3 marks**]

10. Determine

(i) $\int (4x^7 - 3x^2 + 1)\, dx$

[**3 marks**]

(ii) $\int \dfrac{5}{(3x^2 - 5)}\, dx$

[**3 marks**]

(iii) $\int 7x \sin(4x)\, dx$

[**4 marks**]

(iv) $\int 8xe^{-x^2}\, dx$ (change variable to $u = x^2$)

[**4 marks**]

(v) $\int_1^2 \dfrac{2}{x}\, dx$.

[**3 marks**]

11. Solve the differential equation

$$\frac{dy}{dx} = -4y$$

given that $y = -2$ when $x = 0$.

[**3 marks**]

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SECTION B

12. The coordinates of the points A, B and C are $(3, 2, 4)$, $(5, -1, 2)$ and $(7, -1, -3)$ respectively.

(i) Write down the line vectors $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

[3 marks]

(ii) Hence, find to the nearest degree the angle between the lines AB and BC.

[4 marks]

(iii) Find the vector form of the equation of the straight line which passes through the points B and C.

[3 marks]

(iv) Determine the point where the line $\mathbf{r} = (-1, 1, 16) + \mu(2, -2, -4)$ intersects with the line through B and C, where μ is a parameter.

[5 marks]

13. Given

$$y = \frac{3x^2}{(x^2 - 9)}$$

show that

$$\frac{dy}{dx} = -\frac{54x}{(x^2 - 9)^2} .$$

[4 marks]

Sketch the graph of y. Include on your graph the coordinates of all stationary points, the coordinates of any points where the graph crosses the axes and the equations of all the asymptotes.

[11 marks]

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14. Given that $w = 2x^2 - y^2 - 6xy - 4x - 2$ find

$$\frac{\partial w}{\partial x} \ , \ \frac{\partial w}{\partial y} \ , \ \frac{\partial^2 w}{\partial x^2} \ , \ \frac{\partial^2 w}{\partial y^2} \ \text{and} \ \frac{\partial^2 w}{\partial x \partial y} \ .$$

[5 marks]

Find and classify any stationary points of w.

[6 marks]

Find the rate of change of w at the point $(1, 2)$ in the direction of the point $(3, 7)$.

[4 marks]

15. (i) Sketch the region $|z - 2 + 3i| = 4$ on the complex plane where $z = x + iy$.

[3 marks]

(ii) Find two polar forms and two exponential forms for $(1 + 2i)$.

[7 marks]

(iii) Given

$$z = 2\left[\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\right] \quad \text{and} \quad w = 3\left[\cos\left(\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{6}\right)\right]$$

find a polar and an exponential form for z^2 and w/z^3.

[5 marks]

16. (i) Solve the differential equation

$$\frac{dy}{dx} = 7y + \sin(3x) \ .$$

[9 marks]

(ii) Solve the differential equation

$$\frac{d^2y}{dx^2} = -81y$$

given that $y = 3$ and $dy/dx = -4$ when $x = 0$.

[6 marks]