

PAPER CODE NO.
MATH199



THE UNIVERSITY
of LIVERPOOL

SUMMER 2002 EXAMINATIONS

Bachelor of Engineering : Year 1
Bachelor of Science : Year 1
Master of Engineering : Year 1

MATHEMATICS FOR ENGINEERING SCIENCE

TIME ALLOWED : Three Hours

INSTRUCTIONS TO CANDIDATES

You may attempt all questions. All answers to Section A and to the best THREE questions from Section B will be taken into account. Section A carries 55% of the available marks.

Your attention is drawn to the formula list which accompanies this exam paper.



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SECTION A

1. Given $\mathbf{a} = 4\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}$ and $\mathbf{b} = \mathbf{i} + 2\mathbf{j} - 5\mathbf{k}$ find $3\mathbf{a} - \mathbf{b}$. Hence, determine to three decimal places, the magnitude of $3\mathbf{a} - \mathbf{b}$ and to the nearest degree, the angle between $3\mathbf{a} - \mathbf{b}$ and the x -axis.

[4 marks]

2. Find the scalar product of the vectors $5\mathbf{i} + 3\mathbf{j} - 7\mathbf{k}$ and $2\mathbf{i} - 6\mathbf{j} + \mathbf{k}$. Determine, to the nearest degree, the angle between these two vectors.

[3 marks]

3. Differentiate the following with respect to x , simplifying your answer,

$$(i) \quad 7x^2e^{-5x} \quad , \quad (ii) \quad \frac{2x^3}{(3x-4)} .$$

[5 marks]

4. Sketch the graph of $y = 4e^{-5x} - 3$. Include in your graph the coordinates of the points where the graph crosses the x - and y - axes and the equation of the asymptote.

[4 marks]

5. Sketch the level curves $w = -1$, 0 and 1 of the function $w = yx^2$.

[4 marks]

6. Given that $w = x^4e^{-3y}$, find

$$\frac{\partial w}{\partial x} \quad , \quad \frac{\partial w}{\partial y} \quad , \quad \frac{\partial^2 w}{\partial x^2} \quad \text{and} \quad \frac{\partial^2 w}{\partial x \partial y} .$$

[4 marks]

7. Given that $w = 2x^3y + 3x^2y^2$, find the approximate value of w if $x = 2 \pm 0.1$ and $y = 1 \pm 0.1$.

[4 marks]



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8. Express $(3 - 2i)/(1 - 4i)$ in the form $a + ib$ where a and b are real numbers.
[3 marks]

9. Express in terms of the complex variable $z = x + iy$ the interior of the circle centred on $(2, 1)$ with radius 4.
[3 marks]

10. Determine

(i) $\int (8x^3 - 4x + 5) dx$
[3 marks]

(ii) $\int \frac{4}{(3x - 1)(2x + 7)} dx$
[3 marks]

(iii) $\int 3xe^{-2x} dx$
[4 marks]

(iv) $\int 4 \sin x \cos^5 x dx$ (change variable to $u = \cos x$)
[4 marks]

(v) $\int_0^1 2(3x + 1)^4 dx$.
[4 marks]

11. Solve the differential equation

$$\frac{dy}{dx} = \frac{1}{(1 + x^2)}$$

given that $y = 5$ when $x = 0$.
[3 marks]



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SECTION B

12. The coordinates of the points A , B and C are $(-1, 0, 3)$, $(-5, 4, 4)$ and $(3, 2, -4)$ respectively.

(i) Write down the line vectors \overrightarrow{AB} and \overrightarrow{BC} .

[3 marks]

(ii) Hence, find to the nearest degree the angle between the lines AB and BC .

[6 marks]

(iii) Find the vector form of the equation of the straight line which passes through the points B and C .

[3 marks]

(iv) Does the point $(-1, 3, 0)$ lie on this line? Justify your answer.

[3 marks]

13. Given

$$y = \frac{x^2}{(x-4)}$$

show that

$$\frac{dy}{dx} = \frac{x(x-8)}{(x-4)^2}.$$

[4 marks]

Sketch the graph of y . Include on your graph the coordinates of all stationary points, the coordinates of any points where the graph crosses the axes and the equations of all the asymptotes.

[11 marks]

14. Given that $w = x^3 + 6xy + y^3$ find

$$\frac{\partial w}{\partial x}, \quad \frac{\partial w}{\partial y}, \quad \frac{\partial^2 w}{\partial x^2}, \quad \frac{\partial^2 w}{\partial y^2} \quad \text{and} \quad \frac{\partial^2 w}{\partial x \partial y}.$$

[5 marks]

Find and classify the stationary points of w .

[6 marks]

Find the rate of change of w at the point $(4, 3)$ in the outward radial direction.

[4 marks]



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15. (i) Working to three decimal places, find a polar and an exponential form for the complex numbers $(2 - i)$ and $(3 + 2i)$.

Hence working to three decimal places, find a polar and an exponential form for

$$\frac{(3 + 2i)^2}{(2 - i)}.$$

[9 marks]

(ii) Given that $w = 3z^2 - 2z + 5$, where $w = u + iv$, $z = x + iy$ and u , v , x and y are real, find u and v as functions of x and y .

Hence show that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0.$$

[6 marks]

16. (i) Solve the differential equation

$$\frac{dy}{dx} = 2y - 3e^{3x} \sin(4x).$$

[9 marks]

(ii) Solve the differential equation

$$\frac{d^2 y}{dx^2} = 16y$$

given that $y = 4$ and $dy/dx = 2$ when $x = 0$.

[6 marks]