MATH199 Summer 2001

You may attempt all questions. All answers to Section A and to the best THREE questions from Section B will be taken into account. Section A carries 55% of the available marks.

Your attention is drawn to the formula list which accompanies this exam paper.

SECTION A

1. Given $\mathbf{a} = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$ and $\mathbf{b} = 4\mathbf{i} + \mathbf{j} + \mathbf{k}$ find $\mathbf{a} - 2\mathbf{b}$. Hence, find correct to three decimal places the magnitude of $\mathbf{a} - 2\mathbf{b}$ and to the nearest degree, the angle between $\mathbf{a} - 2\mathbf{b}$ and the y-axis. [4 marks]

2. Find the scalar product of the vectors $\mathbf{i} + 2\mathbf{j} + 7\mathbf{k}$ and $3\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$. Find to the nearest degree the angle between these two vectors. [3 marks]

3. Differentiate (i) $4x^2e^{-4x}$, (ii) $\dfrac{2x^3}{(3x^2+4)}$. [5 marks]

4. Sketch the graph of $y = 2e^{3x} - 7$. Include in your graph the coordinates of the points where the graph crosses the x and y axes and the equation of the asymptote. [4 marks]

5. Find and classify the stationary points of the function [5 marks]

$$w = x^2 + 3xy + 2y^2 - x + y - 5 .$$

6. Given that $w = 3x^2\sin(5y)$, find [4 marks]

$$\frac{\partial w}{\partial x} \ , \ \frac{\partial w}{\partial y} \ , \ \frac{\partial^2 w}{\partial x^2} \text{ and } \frac{\partial^2 w}{\partial x \partial y} .$$

7. The number of defects, n, in a material of volume V at temperature T is given by $n = cT^3/V^4$ where c is a constant. The volume fluctuates by $\pm 0.25\%$ and the temperature fluctuates by $\pm 0.5\%$. Find the approximate resultant percentage fluctuation of n. [3 marks]

8. Express $4/(3+2i)^2$ in the form $a + ib$ where a and b are real numbers. [3 marks]

9. Sketch the curve $\arg(z) = \pi/6$ in the xy-plane, where $z = x + iy$ is the complex variable. [3 marks]

10. Evaluate

(i) $\displaystyle\int (x^3 - 2x^2 + 5)\, dx$ [3 marks]

(ii) $\displaystyle\int \frac{7}{\sqrt{(x^2 - 25)}}\, dx$ [3 marks]

(iii) $\displaystyle\int 3x \cos(4x)\, dx$ [4 marks]

(iv) $\displaystyle\int 2x \cos(3x^2 - 4)\, dx$ (change variable to $u = 3x^2 - 4$)

[4 marks]

(v) $\displaystyle\int_0^{\infty} 4e^{-2x}\, dx$ [4 marks]

11. Solve the differential equation

$$\frac{dy}{dx} = 7y$$

given that $y = 3$ when $x = 0$. [3 marks]

SECTION B

12. The coordinates of the points A, B and C are $(1, 2, -1)$, $(3, -4, 2)$ and $(7, -2, 4)$ respectively.

(i) Write down the line vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$. [3 marks]

(ii) Hence, find to the nearest degree the angle between the lines AB and AC. [6 marks]

(iii) Find the vector form of the equation of the straight line which passes through points A and B. [3 marks]

(iv) Show that the point whose coordinates are $(-7, 26, -13)$ lie on this line. [3 marks]

13. Given that

$$y = \frac{x^2}{(x+2)}$$

show that

$$\frac{dy}{dx} = \frac{x(x+4)}{(x+2)^2} .$$

[5 marks]

Sketch the graph of y. Include on your graph the coordinates of all stationary points, the coordinates of any points where the graph crosses the axes and the equations of all the asymptotes. [10 marks]

14. Sketch the level curves $w = -1$, 0, 1 of the function $w = y - 3x^2$. [5 marks]

(i) Find the rate of change of w at the point $(2, 1)$ and in the outward radial direction. [5 marks]

(ii) Find the rate of change of w at the point $(3, 2)$ and in the direction towards $(4, 5)$. [5 marks]

15. Given the harmonic function $V = 4\cos(3t) + 2\sin(3t)$, write down its amplitude and period. [2 marks]

Sketch V as a function of t. Show on your graph the maximum and minimum values of V and the period of V. [2 marks]

Express V as a cosine harmonic, evaluating the phase angle to 3 decimal places. [4 marks]

Hence find the values of t at which V has its maximum value. [3 marks]

Express V in terms of a complex harmonic function. Write down the complex amplitude of this complex harmonic function. [2 marks]

Write down the differential equation satisfied by V. [2 marks]

16. (i) Solve the differential equation

$$\frac{dy}{dx} = 2y + \sin(3x) .$$

[8 marks]

(ii) Solve the differential equation

$$\frac{d^2y}{dx^2} = -25y$$

given that $y = 17$ and $dy/dx = 25$ when $x = 0$. [7 marks]