PAPER CODE NO.
MATH 192

THE UNIVERSITY
of LIVERPOOL

MAY EXAMINATIONS 2007

Bachelor of Engineering : Year 1
Bachelor of Engineering : Year 2
Master of Engineering : Year 1

MATHEMATICS II FOR ELECTRICAL ENGINEERS

TIME ALLOWED : Three Hours

INSTRUCTIONS TO CANDIDATES

Full marks will be awarded for complete answers to ELEVEN questions of which

- NINE answers must be from **Section A** and
- TWO from **Section B**
(only the best 2 answers from Section B will be taken into account).

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SECTION A

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1. [4 marks]

Evaluate the following indefinite integrals:

(a)

$$I_1 = \int (x + 2007)^{2007} dx;$$

(b)

$$I_2 = \int (\sin(\pi x) + \frac{1}{x+1}) dx.$$

2. [5 marks]

Evaluate the following definite integral:

$$\int_0^{(\frac{\pi}{2})^{1/3}} 6x^2 \cos(x^3) dx.$$

3. [5 marks]

Firstly complete the following partial fraction

$$\frac{13\,x^2 + 11\,x + 12}{(x+1)\,(x+2)\,(x+3)} = \frac{a}{x+1} + \frac{b}{x+2} + \frac{c}{x+3}$$

and secondly evaluate the definite integral

$$\int_0^1 \frac{13\,x^2 + 11\,x + 12}{(x+1)\,(x+2)\,(x+3)} dx.$$

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4. [5 marks]

Find a function $u(x,y)$ such that

$$\begin{cases} \dfrac{\partial u}{\partial x} = 5x^4 e^{-2y} + 4x^3, \\ \dfrac{\partial u}{\partial y} = -2x^5 e^{-2y} - 2y. \end{cases}$$

5. [5 marks]

Given the function, representing a three-dimensional surface,

$$z = z(x,y) = 5\,x^2 - 7\,xy + 3\,y^2 + x + 2\,y - 9$$

find the equation of its tangent plane at the point $(x_0, y_0, z_0) = (1, 2, -1)$.

6. [5 marks]

Find the general solution of the following differential equation

$$\frac{dy}{dx} = \frac{y\,(x)}{x+1} - x.$$

Further find the specific solution satisfying the condition $y(0) = 3$.

7. [5 marks]

Solve the following second order differential equation:

$$\frac{d^2y}{dx^2} - 9\,\frac{dy}{dx} + 14\,y\,(x) = 0.$$

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8. **[5 marks]**

Given three position vectors (relative to the origin)

$$\mathbf{a} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}, \quad \mathbf{b} = -\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}, \quad \mathbf{c} = 2\mathbf{i} + 4\mathbf{j} + 5\mathbf{k},$$

i) First find the angle θ between $\mathbf{a}$ and $\mathbf{b}$;

ii) Then find the vector product of $\mathbf{b}\times\mathbf{c}$;

iii) Finally compute the volume of the parallelepiped formed by these 3 vectors starting from the origin.

9. **[5 marks]**

Given the matrix A and the vector b as follows:

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ 2 & 1 & -1 \end{pmatrix}, \quad b = \begin{pmatrix} 7 \\ 9 \\ 1 \end{pmatrix},$$

i) Evaluate $det(A)$;

ii) Solve the linear system $Ax = b$ for x.

SECTION B

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10. **[18 marks]**

i) Evaluate

$$\int_{\pi/2}^{\pi} t^2 \cos(t) dt.$$

ii) Using the equalities $\sin(A + B) = \sin(A) \cos(B) + \cos(A) \sin(B)$ and $\cos 2t = 2\cos^2 t - 1$ or otherwise, first prove that

$$\sin(3t) + 4\sin^3(t) = 3\sin(t).$$

Further evaluate

$$I_{10} = 3\int_0^{\pi/2} \sin^3(t) dt.$$

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11. [18 marks]

A laboratory in Department of DMS installed a LC circuit with sinusoidal input. The current flow $I(t)$ at time t obeys the following differential equation:

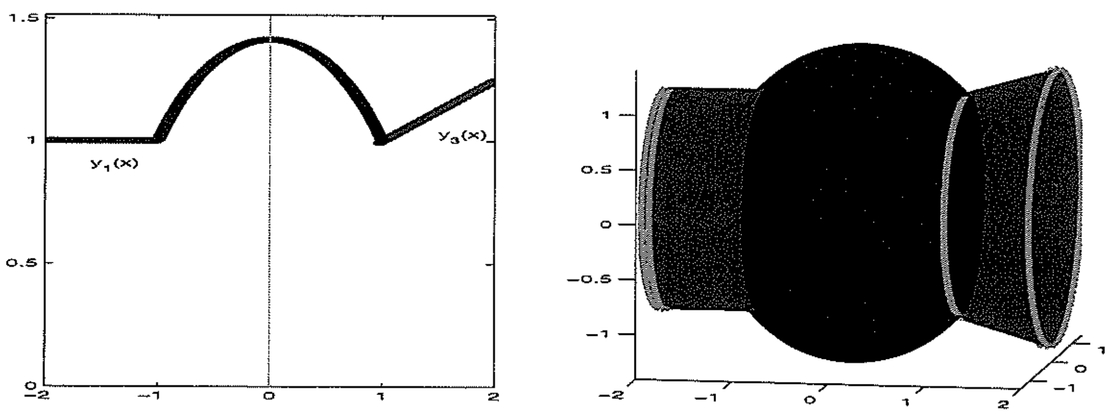
$$\frac{d^2 I}{dt^2} - \frac{dI}{dt} = 2\,I\,(t) + \frac{1}{50}\,\cos\,(2\,t)\,.$$

Find the specific solution satisfying the conditions

$$I(0) = 1/2, \qquad \left.\frac{dI}{dt}\right|_{t=0} = -1/4.$$

12. [18 marks]

A semiconductor device is manufactured with built-in acoustic functionality, having a physical shape as shown below (not to scale) where the right plot shows the shape of the product and the left plot shows the function $y = y(x)$ used to rotate 360^o degrees along the horizontal x-axis to make the product:



Here the function $y = y(x)$ is defined by

$$y(x) = \begin{cases} y_1(x) = 1, & -2 \le x < -1, \\ y_2(x) = \sqrt{2 - x^2}, & -1 \le x < 1, \\ y_3(x) = \frac{x}{4} + \frac{3}{4}, & 1 \le x \le 2. \end{cases}$$

Compute the surface area of the product (excluding the 2 circular areas at both ends).
Hint. Clearly the surface is consisted of 3 *separate surfaces.*

13.

[18 marks]

Given the matrix A and the vector b as follows:

$$A = \begin{pmatrix} 9 & 0 & 1 & 0 \\ 1 & -1 & 0 & 2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 3 \end{pmatrix}, \quad b = \begin{pmatrix} 64 \\ 10 \\ -1 \\ 7 \end{pmatrix},$$

i) First verify that the matrix C of co-factors has these entries: $C_{11} = C_{12} = -4$, $C_{13} = C_{14} = C_{21} = C_{23} = C_{24} = 0$ and $C_{22} = 36$.

Then compute $det(A), C^T$ and A^{-1}.

ii) Find the solution x to the linear system $Ax = b$.