

PAPER CODE NO.  
MATH181



THE UNIVERSITY  
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**Resit 2007 EXAMINATIONS**

Bachelor of Science : Year 1  
Master of Chemistry : Year 1  
Master of Earth Sciences : Year 1  
Master of Physics : Year 1

**METHODS**

TIME ALLOWED : Two Hours and a Half

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**INSTRUCTIONS TO CANDIDATES**

Answer ALL questions in Section A and THREE questions from Section B. Section A carries 55% of the available marks.

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SECTION A

1. A function is defined by

$$f(x) = 2 \sin x$$

for  $0 \leq x \leq 4\pi$ . Sketch this function.

[3 marks]

2. What is a one-to-one function? A function is given by

$$y = f(x) = \frac{2x + 3}{2x - 5}.$$

What is the domain and range? Find  $f^{-1}(x)$  given that the function  $f(x)$  is one-to-one.

[5 marks]

3. Differentiate the following with respect to  $x$

$$(i) \quad 3x^7 e^{-x^2} \quad , \quad (ii) \quad \frac{2x^3}{(x+5)^2} \quad , \quad (iii) \quad \cos(3x) \sinh^4(2x) .$$

[9 marks]

4. Suppose that two variables satisfy the equation

$$x^4 - y^3 x + \frac{6}{x} \sin(y) = 8.$$

Find implicitly  $dy/dx$  in terms of  $y$  and  $x$ .

[5 marks]

5. Determine the following indefinite integrals

$$(i) \quad \int \left( x - \frac{1}{x} \right) dx \quad , \quad (iii) \quad \int \sinh^2(5x) dx .$$

[5 marks]

6. Evaluate

$$(i) \quad \int_0^\pi x \sin^2 x dx \quad , \quad (iii) \quad \int_{-\infty}^\infty x e^{-4x^2} dx .$$

[5 marks]



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7. (i) Evaluate the sum

$$\sum_{k=0}^3 5^k.$$

(ii) State the ratio test. Does the sum

$$\sum_{k=0}^{\infty} \left( \frac{3^k}{k!} \right)$$

converge?

[5 marks]

8. If  $z_1 = 4 + 3i$  and  $z_2 = 5 + 2i$  determine in the form  $a + ib$

$$(i) \quad z_1 + 2z_2 \quad , \quad (ii) \quad z_1 \bar{z}_2 \quad , \quad (iii) \quad |z_1 z_2| .$$

What is  $\arg(z_1 z_2)$ ?

[5 marks]

9. Suppose that

$$g(x, y, z) = (x^3 + y^4 - z^2 x^2).$$

What are  $\partial g / \partial x$ ,  $\partial g / \partial y$ ,  $\partial^2 g / \partial x^2$  and  $\partial^2 g / \partial x \partial z$ ?

[5 marks]

10. Consider the function  $\cosh^2(3x) + \sinh^2(3x)$ . Obtain the Maclaurin series expansion of this function up to and including the term  $x^4$ .

[4 marks]

11. A class of five students obtain the marks 34, 50, 55, 70, and 85 in an exam. What is the mean mark and the variance of the marks for this exam.

[4 marks]



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SECTION B

12. (i) Evaluate the definite integral

$$\int_3^4 \frac{x^2}{(x-2)(x+2)} dx.$$

[6 marks]

(ii) Evaluate the integral

$$\int (x^2 + y^2) \sqrt{dx^2 + dy^2}$$

on the curve  $x = \cos \theta$  and  $y = \sin \theta$  between  $\theta = 0$  and  $\theta = \pi$ .

[3 marks]

(iii) Using a suitable substitution or otherwise evaluate the indefinite integral

$$\int \frac{1}{(e^{2x} + 5e^x)} dx.$$

[6 marks]

13. (i) By using polar coordinates or otherwise, integrate the function

$$f(x, y) = (x^2 + y^2)^{5/2}$$

over the area enclosed by the curve  $x^2 + y^2 = 4$ .

[6 marks]

(ii) Evaluate the integral

$$\int_A (y^4 \sin x + y^2 \sin 2x) dx dy$$

where the area  $A$  is bounded by the lines  $y = 0$ ,  $x = 0$  and  $y = \cos x$ .

[9 marks]



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14. (i) Find the cube roots of the number 8 and plot these in the complex plane.

[6 marks]

(ii) Use Eulers formula to show that

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta.$$

[6 marks]

(iii) Using this result determine

$$\int \cos 5\theta \cos 3\theta d\theta.$$

[3 marks]

15. (i) Sketch the graph

$$y = \frac{x + 2}{x - 1}$$

in detail.

[6 marks]

(ii) Using the Maclaurin series expansion to the first three terms of  $\sin(x^3)$ , compute the approximate value of

$$\int_0^1 \sin(x^3) dx.$$

[5 marks]

(iii) Evaluate the limit

$$\lim_{x \rightarrow 0} \frac{\sqrt{3 - 2 \cos x} - 1}{x^2}.$$

[4 marks]



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16. (i) How many distinct ways are there of arranging the word “MISSISSIPPI”?  
[3 marks]

(ii) Find the number of ways a magician can choose six of the ten white rabbits that are sat in his top hat.  
[4 marks]

(iii) A company orders 20 computers from one manufacturer  $A$  and 20 computers from manufacturer  $B$  that are totally independent. Manufacturer  $A$  finds that 4% of their computers have defective memory, whilst manufacturer  $B$  finds that 6% of theirs suffer similarly. These memory problems occur randomly in the manufacturing process.

Determine the probability that in the order to the company exactly 3 computers from manufacturer  $B$  have defective memory. Also determine the probability that only one computer in the order of 40 computers has defective memory.

[8 marks]