

Candidates should answer the **WHOLE** of Section A and **THREE** questions from Section B. Section A carries 55% of the available marks.

## SECTION A

1. A function is defined by

$$f(x) = -|x + 2| .$$

Sketch the function  $f(x)$ .

[3 marks]

2. State the domain of the function

$$y = f(x) = \frac{(x + 1)}{(x - 3)} .$$

Given that  $f$  is a one-to-one function, find  $f^{-1}(x)$  .

[3 marks]

3. Differentiate with respect to  $x$

$$(i) \quad x^4 \sinh x , \quad (ii) \quad (1 + x^3)^{10} , \quad (iii) \quad \sqrt{1 + e^{\sin x}} .$$

[9 marks]

4. Given that

$$y^4 - 2x^2y^2 + x\sqrt{1+x} = 4 ,$$

find  $\frac{dy}{dx}$  in terms of  $x$  and  $y$ .

[4 marks]

5. Determine the following indefinite integrals

$$(i) \quad \int (e^{4x} + x^3) dx , \quad (ii) \quad \int \frac{x}{2x + 5} dx .$$

[6 marks]

6. Evaluate

$$(i) \quad \int_0^1 \frac{x}{(x^2 + 1)} dx , \quad (ii) \quad \int_0^\pi x \cos x dx .$$

[7 marks]

7. (i) Evaluate the sum

$$2 \sum_{k=0}^{10} 3^k .$$

(ii) Find the limit of the sequence

$$y_n = \frac{2n^2 + 1}{4n^2 + 3} ,$$

as  $n \rightarrow \infty$ .

[5 marks]

8. Given that  $z_1 = 1 + i$  and  $z_2 = 2 + i$  determine, in the form  $a + ib$ ,

$$(i) \quad 2z_1 - 3z_2 , \quad (ii) \quad z_1 z_2 , \quad (iii) \quad |z_1| .$$

Convert the complex number  $2 e^{i\frac{\pi}{4}}$  into the form  $a + ib$ .

[5 marks]

9. Given that

$$f(x, y) = x + 2x^2y^2 + x\sqrt{1+y} ,$$

find  $f_x, f_y, f_{xy}, f_{xx}$  and  $f_{yy}$ .

[5 marks]

10. Obtain the Maclaurin series expansion for  $(1 - x^2) \cos 2x$  up to and including the term in  $x^4$  .

[4 marks]

11. A class of four students obtained the marks: 35, 80, 50, and 60. What is the mean and variance of the marks?

[4 marks]

## SECTION B

12. (a) Find the constants  $A$  and  $B$  for which

$$\frac{1}{(x-2)(x+2)} = \frac{A}{x-2} + \frac{B}{x+2},$$

is true.

[3 marks]

- (b) Evaluate

$$\int_0^1 \frac{1}{(x^2-4)} dx,$$

using the result in part (a) of this question.

[3 marks]

- (c) Use a suitable substitution to determine the indefinite integral

$$\int \frac{1}{e^{2x} - 3e^x} dx.$$

[6 marks]

- (d) Determine whether the integrand below is even, odd, or neither, and use that observation to evaluate the integral

$$\int_{-5}^5 (x^3 + x^6 \sin x + \cos^3 x \tan x) dx.$$

[3 marks]

13. (a) Evaluate

$$\int_0^1 \left( \int_0^2 (x^2 y + xy^2) dy \right) dx.$$

[5 marks]

- (b) Using polar coordinates, or otherwise, integrate

$$f(x, y) = (x^2 + y^2)^{\frac{3}{2}}$$

over the area enclosed by the curve  $x^2 + y^2 = 4$  and with the condition  $y > 0$ .

[5 marks]

- (c) Evaluate

$$\int \int_A (6y^2 \cos x) dx dy,$$

where  $A$  is the region of the  $xy$ -plane bounded by the lines  $y = \sin x$ ,  $x = \frac{\pi}{2}$  and the  $x$  axis.

[5 marks]

14. (a) Find in polar form all the roots of the equation

$$z^3 = -8i ,$$

and draw a diagram showing their position in the complex plane.

[4 marks]

- (b) Use  $\sin \theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$  to show that

$$\sin^5 \theta = \frac{1}{16} (\sin 5\theta - 5 \sin 3\theta + 10 \sin \theta) .$$

[4 marks]

- (c) Using the result in part (b) determine

$$\int \sin^5 x \, dx .$$

[3 marks]

- (d) Write

$$\sqrt{1+i} ,$$

in the form  $a + ib$ .

[4 marks]

15. (a) Sketch the graph

$$y = \frac{x+4}{x-3} .$$

[5 marks]

- (b) Use the power series expansion of  $\cos \sqrt{x}$  up to order  $x^3$  to obtain an approximate estimate for the integral

$$\int_0^1 \cos \sqrt{x} \, dx ,$$

correct to three decimal places.

[5 marks]

- (c) Given that the slope for  $\sec^{-1} x$  is positive for  $x > 0$ , show that

$$\frac{d \sec^{-1} x}{dx} = \frac{1}{x\sqrt{x^2-1}} .$$

[5 marks]

16. (a) How many distinct arrangements are there of the word “London”?

[3 marks]

(b) A committee of seven members is to be divided into three subcommittees, two of which are size 2, and one is size 3. How many ways can this be done?

[3 marks]

(c) A courier service buys 8 cars from a garage. There is a problem with 6% of the cars sold from the garage. The occurrence of the problem with the cars is random.

Determine the probability that for the cars from the garage:

- (i) exactly three cars have a problem;
- (ii) less than two cars have a problem.

[5 marks]

(d) If  $N$  people tell you their birthday. What is the value of  $N$  such that the probability of meeting a person who shares your birthday is 0.5? Ignore leap years.

[4 marks]