Candidates should answer the WHOLE of Section A and THREE questions from Section B. Section A carries 55% of the available marks.

SECTION A

1. A function is defined by

$$f(x) = -|x| \text{ for } -1 < x \leq 1$$

and for other values of x by

$$f(x+2) = f(x)$$

Sketch $f(x)$ between -1 and 3. [3 marks]

2. State the domain of the function

$$y = f(x) = \frac{(x+2)}{(2x+3)} \ .$$

Given that f is a one-to-one function, find $f^{-1}(x)$.

[3 marks]

3. Differentiate with respect to x

$$(i) \quad \cos(5x)\sin^2(x) \qquad (ii) \quad (e^{3x}+2)^3 \qquad (iii) \quad \frac{\cosh x}{x^2} \ .$$

[9 marks]

4. Given that

$$\frac{1}{x^2} - 3xy^2 + x\sin(y) = 6$$

find $\frac{dy}{dx}$ in terms of x and y. [4 marks]

5. Determine the following indefinite integrals

$$(i) \quad \int (\cos(3x) + x^2)\, dx \qquad (ii) \quad \int \frac{1}{3x^2+1}\, dx \ .$$

[6 marks]

6. Evaluate

$$(i) \quad \int_0^1 \frac{1}{(2x+3)^2}\, dx \qquad (ii) \quad \int_0^2 xe^x\, dx \ .$$

[7 marks]

7. (i) Evaluate the sum

$$(i) \quad \sum_{k=0}^{6}(5(2^k) - 3k)$$

(ii) Use the ratio test to show that the sum

$$\sum_{k=1}^{\infty}\left(\frac{2^k}{k!}\right)$$

is convergent. [5 marks]

8. Given that $z_1 = 1 + 3i$ and $z_2 = -3 + i$ determine, in the form $a + ib$,

$$(i) \quad 2z_1 - 3z_2 \ , \qquad (ii) \quad z_1\overline{z}_2 \ , \qquad (iii) \quad \frac{z_1}{z_2} \ , \qquad (iv) \quad \mid z_1 \mid$$

Compute $\text{Arg}(z_1)$ in degrees.

[5 marks]

9. Given that

$$f(x, y) = xy^2 + x\sin(x + y)$$

find f_x, f_y, f_{xy}, f_{xx} and f_{yy}. [5 marks]

10. Obtain the Maclaurin series expansion for $\sin(2x) - \cos(2x)$ up to and including the term in x^4 . [4 marks]

11. If 500 people select a number at random between 1 and 200, what is the probability that three people selected the number 181. [4 marks]

SECTION B

12. (a) Evaluate

$$\int_0^1 \frac{x^2}{(x+1)(x+2)}\,dx\ .$$

[6 marks]

(b) Evaluate the integral

$$\int 2xy\,ds$$

along the curve $x = \cos(t)$ and $y = \sin(t)$ from $t = 0$ to $t = \pi/4$.

[3 marks]

(c) Use a suitable substitution to determine the indefinite integral

$$\int \frac{\cos(x)\sin(x)}{(1+3\cos(2x))^2}\,dx\ .$$

[6 marks]

13. (a) Using polar coordinates, or otherwise, integrate

$$f(x,y) = 1 + (x^2 + y^2)$$

over the area enclosed by the curve $y^2 + x^2 = 1$.

[6 marks]

(b) Evaluate

$$\int\int_A (x^2 y + 2)\,dxdy$$

where A is the region of the xy-plane bounded by the lines $y = 0, y = 1 - 2x$ and $y = 1 - x$. [9 marks]

14.

(a) Find in polar form all the roots of the equation

$$z^3 = 1 - i$$

and draw a diagram showing their position in the complex plane.

[6 marks]

(b) Use Euler's formulae to show that

$$\sin(x \pm y) = \sin(x)\cos(y) \pm \cos(x)\sin(y) \ .$$

[6 marks]

(c) Hence determine

$$\int \sin(3x)\cos(5x)dx \ .$$

[3 marks]

15. (a) Given that

$$\int e^{ax}dx = \frac{e^{ax}}{a}$$

determine

$$\int x^2 e^{ax}dx$$

either by integration by parts or via differentiation with respect to the parameter a. [6 marks]

(b) Use the first three terms in the Maclaurin series expansion of $\sin(x^2)$ to compute

$$\int_0^1 \sin(x^2)dx \ .$$

[5 marks]

(c) Evaluate using the Maclaurin series for $\cos(x)$

$$\lim_{x\to 0} \frac{\sqrt{(5-\cos(x))}-2}{x^2}.$$

[4 marks]

16. (a) How many distinct arrangements are there of the word "statistics"?

[3 marks]

(b) Find the number of ways a research worker can choose eight of the 12 largest cities in the UK for a survey of air pollution.

[4 marks]

(c) A company orders 10 computers from one manufacturer (X) and 10 computers from manufacturer (Y). The two manufacturers are totally independent, but they both occasionally produce computers with defective memory. Manufacturer X finds that 3 % of the computers shipped to customers have errors with defective memory. Manufacturer Y finds that 5 % of their computers shipped to customers have errors with defective memory. These memory errors occur randomly in the manufacturing process.

Determine the probability that in the order to the company

(i) exactly 2 computers from manufacturer X have memory errors;

(ii) that only one computer in the order of 20 computers has defective memory.

[8 marks]