Candidates should answer the WHOLE of Section A and THREE questions from Section B. Section A carries 55% of the available marks.

SECTION A

1. Sketch the graph of

$$y = 6 - |x| \ .$$

[3 marks]

2. State the domain of the function

$$y = f(x) = \frac{x}{x+1} \ .$$

Given that f is a one-one function, find $f^{-1}(x)$.

[3 marks]

3. Differentiate with respect to x

$$(i) \quad e^x \sin^2 x, \qquad (ii) \quad (x^3+1)^2, \qquad (iii) \quad \frac{\sin x}{x}.$$

[8 marks]

4. Given that

$$x^3 + 4x^2y + xy^2 = 6$$

find $\frac{dy}{dx}$ in terms of x and y. [4 marks]

5. Evaluate the following indefinite integrals

$$(i) \quad \int (4x^{-3} + x^{-1})\, dx, \qquad (ii) \quad \int \frac{x}{x^2+1}\, dx \ .$$

[5 marks]

6. Evaluate

$$(i) \quad \int_0^1 (2x-1)^3\, dx \qquad (ii) \quad \int_0^{\pi/2} x \sin x\, dx \ .$$

[9 marks]

7. Evaluate the sums

$$(i) \quad \sum_{k=0}^{8}(3+2k), \qquad (ii) \quad \sum_{k=1}^{5}\left(\frac{1}{4}\right)^{k}.$$

[4 marks]

8. Given that $z_1 = 1 + i$ and $z_2 = -2 - 3i$ determine, in the form $a + ib$,

$$(i) \quad 2z_1 + 2z_2 \;, \qquad (ii) \quad z_1 z_2 \;, \qquad (iii) \quad \frac{1}{z_2} \;, \qquad (iv) \quad z_1^2 \;.$$

Express z_1 in polar form.

[6 marks]

9. Given that

$$f(x, y) = xye^{x}$$

find f_x, f_y, f_{xx} and f_{xy} . [5 marks]

10. Obtain the Maclaurin series expansion for $x \sin 2x$ up to and including the term in x^4 . [4 marks]

11. The letters *EILLOOPRV* are written on 9 otherwise identical cards. The cards are chosen randomly one at a time and placed in order. What is the probability that they spell *LIVERPOOL*? [4 marks]

SECTION B

12.

(a) Find constants A and B such that

$$\frac{x}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2}.$$

Hence evaluate

$$\int_{-1}^{0} \frac{x}{(x-1)(x-2)}\, dx\ .$$

[6 marks]

(b) Evaluate the definite integral

$$\int_{0}^{\infty} x^2 e^{-x}\, dx\ .$$

[6 marks]

(c) Use a suitable substitution to evaluate the indefinite integral

$$\int \frac{\sin x}{(1+\cos x)^2}\, dx\ .$$

[3 marks]

13.

(a) Find the area enclosed between the curve $y = x^3$ and the straight line $y = x$ and with $x \geq 0$

[6 marks]

(b) Evaluate

$$\int\int_A xy\, dxdy$$

where A is the region of the xy-plane bounded by the lines $x = 0, y = 0$ and $y = 1 - x$.

[9 marks]

14.

(a) Find in polar form all the roots of the equation

$$z^4 = -16i$$

and draw a diagram showing their position in the complex plane.

[6 marks]

(b) Use de Moivre's theorem to show that

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta \ .$$

[6 marks]

(c) Find all the roots of the equation

$$z^2 + z + 1 = 0$$

expressing each of them in the form $a + ib$.

[3 marks]

15.

(a) Given that

$$y = f(x) = \frac{1}{(x-1)(x-3)}$$

find $f'(x)$ and $f''(x)$ *either* by first expanding $f(x)$ in partial fractions and then differentiating *or* by differentiating directly.

Show that the graph of $y = f(x)$ has one local maximum and determine its coordinates. Sketch the graph of $y = f(x)$ indicating any asymptotes.

[11 marks]

(b) Determine

$$\lim_{x\to 0} \frac{\sqrt{(1+x)} - 1}{x} .$$

[4 marks]

16.

(a) A club has 12 members of which 8 are men and 4 are women. Determine the number of ways in which a committee of 4 can be chosen. Also find the number of ways that the committee can be chosen if it includes equal numbers of men and women.

[7 marks]

(b) A fridge manufacturer finds that 2% of the fridges produced are defective. These defects occur randomly in the manufacturing process.

Determine the probability that in a sample of 10 fridges

(i) exactly 1 fridge is defective;

(ii) no fridges are defective;

(iii) fewer than 3 fridges are defective.

[8 marks]