Candidates should answer the WHOLE of Section A and THREE questions from Section B. Section A carries 55% of the available marks.

A formula sheet is attached.

SECTION A

1. Sketch the graph of

$$y = |x+1| - 2 \ .$$

[3 marks]

2. State the domain of the function

$$y = f(x) = \frac{1}{x+3} \ .$$

Given that f is a one-one function, find $f^{-1}(x)$.

[3 marks]

3. Differentiate with respect to x

$$(i) \quad x^2 \sin^2 x, \qquad (ii) \quad (x^4 + x^2 + 1)^{-2}, \qquad (iii) \quad \frac{\sin x}{x^2+2}.$$

[8 marks]

4. Given that

$$x^2 + 4xy^2 + 2y^3 = 7$$

find $\frac{dy}{dx}$ in terms of x and y. [4 marks]

5. Evaluate the following indefinite integrals

$$(i) \quad \int (5x^4 + \sinh x) dx, \qquad (ii) \quad \int (3-2x)^3 dx \ .$$

[5 marks]

6. Evaluate

$$(i) \quad \int_0^1 xe^{2x} \, dx \qquad (ii) \quad \int_0^2 \frac{dx}{(x+1)^2} \ .$$

[9 marks]

7. Evaluate the sums

$$(i)\quad \sum_{k=1}^{10}(2+7k), \qquad (ii)\quad \sum_{k=0}^{12}\left(\frac{1}{2}\right)^k .$$

[4 marks]

8. Given that $z_1 = -1 + i$ and $z_2 = 2 + 3i$ determine, in the form $a + ib$,

$$(i)\quad 3z_1 + 2z_2, \qquad (ii)\quad z_1 z_2, \qquad (iii)\quad \frac{z_1}{z_2} .$$

Show that $(z_1)^2 = -2i$. Hence, or otherwise, find $(z_1)^9$ in the form $a+ib$.

[6 marks]

9. Given that

$$f(x, y) = x^2 y \cos x$$

find f_x, f_y, f_{xx} and f_{xy} . [5 marks]

10. Obtain the Maclaurin series expansion for $(x^2+1)e^x$ up to and including the term in x^3. [4 marks]

11. The letters *AEGIINNORTT* are written on 11 otherwise identical cards. The cards are chosen randomly one at a time and placed in order. What is the probability that they spell *INTEGRATION*? [4 marks]

SECTION B

12.

(a) Find constants A and B such that

$$\frac{(x+3)}{(x+1)(x+5)} = \frac{A}{x+1} + \frac{B}{x+5}.$$

Hence evaluate

$$\int_0^2 \frac{(x+3)}{(x+1)(x+5)}\, dx\ .$$

[6 marks]

(b) Evaluate the indefinite integral

$$\int x^2 \sin 5x \, dx\ .$$

[6 marks]

(c) Use the substitution $u = \sin x$ to evaluate

$$\int \cos x \sin^4 x \, dx\ .$$

[3 marks]

13.

(a) Find the area enclosed by the parabola $y = x^2$ and the straight line $y = x + 2$.

[6 marks]

(b) Evaluate

$$\int\int_A (2x + 2y) dxdy$$

where A is the region of the xy-plane bounded by the lines $x = 2, y = 1$ and $y = x + 1$.

[9 marks]

14.

(a) Find in polar form all the roots of the equation

$$z^3 = 64i$$

and draw a diagram showing their position in the complex plane.

[7 marks]

(b) Use de Moivre's theorem to show that

$$\cos 3\theta = 4\cos^3\theta - 3\cos\theta \ .$$

Hence or otherwise evaluate

$$\int_0^{\frac{\pi}{2}} \cos^3\theta d\theta \ .$$

[8 marks]

15.

(a) Given that

$$y = f(x) = x + 2 + \frac{1}{x+1}$$

find $f'(x)$ and $f''(x)$.

Show that the graph of $y = f(x)$ has one local maximum and one local minimum and determine their coordinates. Sketch the graph showing any asymptotes.

[11 marks]

(b) Determine

$$\lim_{x \to 0} \frac{1 - e^{2x}}{x}.$$

[4 marks]

16.

(a) A swimming club has 14 members of which 9 are men and 5 are women. Determine the number of ways in which a committee of 4 can be chosen. Also find the number of ways that the committee can be chosen if it includes at least one woman.

[7 marks]

(b) A furniture manufacturer finds that 5% of the chairs produced are defective. These defects occur randomly in the manufacturing process.

Determine the probability that in a sample of 6 chairs

(i) exactly 1 chair is defective;

(ii) fewer than 3 chairs are defective.

[8 marks]