Instructions to candidates

Candidates should answer the WHOLE of Section A and THREE questions from Section B. Section A carries 55% of the available marks.

A formula sheet is attached.

SECTION A

1. Sketch the graph of

$$y = 1 - |x - 1| \ .$$

[3 marks]

2. State the domain of the function

$$y = f(x) = \frac{x}{x+1} \ .$$

Given that f is a one-one function find $f^{-1}(x)$.

[3 marks]

3. Differentiate with respect to x

$$(i) \quad e^x \sin 2x, \qquad (ii) \quad (x^2 + x + 1)^3, \qquad (iii) \quad \frac{2 - x^2}{x^3 + 1}.$$

[8 marks]

4. Given that

$$x^3 + 3x^2y + y^3 = 5$$

find $\frac{dy}{dx}$ in terms of x and y. [4 marks]

5. Evaluate the following indefinite integrals

$$(i) \quad \int (3x^2 + 4x^{-3}) dx, \qquad (ii) \quad \int x \cos 4x \, dx \ .$$

[6 marks]

6. Evaluate

$$(i) \quad \int_0^1 (1 - 2x)^4 \, dx \qquad (ii) \quad \int_1^2 \frac{x \, dx}{x^2 + 4} \ .$$

[8 marks]

7. Evaluate the sums

$$(i) \quad \sum_{k=0}^{8}(3+5k), \qquad (ii) \quad \sum_{k=1}^{\infty}\left(\frac{2}{7}\right)^{k}.$$

[4 marks]

8. Given that $z_1 = 1 + i$ and $z_2 = 3 + 2i$ determine, in the form $a + ib$,

$$(i) \quad 3z_1 - 2z_2, \qquad (ii) \quad z_1 z_2, \qquad (iii) \quad \frac{z_1}{z_2} \ ,$$

Express z_1 in polar form. Hence, or otherwise, find $(z_1)^6$.

[6 marks]

9. Given that

$$f(x, y) = xy \sin x$$

find f_x, f_y, f_{xx} and f_{xy} . [5 marks]

10. Find the Maclaurin series expansion for $(x+3)\sin x$ up to and including the term in x^4. [4 marks]

11. The letters $FIIINNTY$ are written on 8 otherwise identical cards. The cards are chosen randomly one at a time and placed in order. What is the probability that they spell $INFINITY$? [4 marks]

SECTION B

12.

(a) Find constants A and B such that

$$\frac{(x+6)}{(x+2)(x+4)} = \frac{A}{x+2} + \frac{B}{x+4}.$$

Hence evaluate

$$\int_1^2 \frac{(x+6)}{(x+2)(x+4)}\,dx\ .$$

[6 marks]

(b) Evaluate the indefinite integrals

$$(i)\quad \int x^2 e^{-2x} dx, \qquad (ii)\quad \int x\sqrt{(3x^2+1)}dx\ .$$

[9 marks]

13.

(a) Find the area enclosed by the parabola $y = 3 + 2x - x^2$ and the x-axis.

[6 marks]

(b) Sketch the region A of the xy-plane bounded by the lines $x = 3, y = 2$ and $x + y = 3$.

Evaluate

$$\int\int_A 2xy\,dxdy\ .$$

[9 marks]

14.

(a) Show that one root of the equation

$$z^4 = 16i$$

is

$$z = 2(\cos\frac{\pi}{8} + i\sin\frac{\pi}{8}) \; .$$

Find the other roots of the equation in polar form.

Draw a diagram showing the position of these roots in the complex plane.

[7 marks]

(b) Use de Moivre's theorem to show that

$$\cos 4\theta = \cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta$$

and

$$\sin 4\theta = 4\cos^3\theta\sin\theta - 4\cos\theta\sin^3\theta \; .$$

[8 marks]

15.

(a) Given that

$$y = f(x) = x^2 + \frac{2}{x}$$

find $f'(x)$ and $f''(x)$.

Show that the graph of $y = f(x)$ has one local minimum and determine its coordinates. Also show that the graph has a point of inflection and a vertical asymptote. Sketch the graph.

[11 marks]

(b) Determine

$$\lim_{x\to 0} \frac{\sin x - x}{x^3}.$$

[4 marks]

16.

(a) A tennis club has 13 members of which 9 are men and 4 are women. Determine the number of ways in which a committee of 4 can be chosen. Also find the number of ways that the committee can be chosen if it includes at least one man. [7 marks]

(b) A pencil manufacturer finds that 3% of the pencils produced are defective. These defects occur randomly in the manufacturing process.

Determine the probability that in a sample of 7 pencils

(i) exactly 2 pencils are defective;

(ii) fewer than 3 pencils are defective.

[8 marks]