Instructions to candidates

Candidates should answer the WHOLE of Section A and THREE questions from Section B. Section A carries 55% of the available marks.

A formula sheet is attached.

SECTION A

1. Sketch the graph of

$$y = |x - 2| + x.$$

[3 marks]

2. In each of the following cases state whether the given function is even, odd or neither even nor odd:

$$(i) \quad x^4 + x^2, \qquad (ii) \quad x^3 \cos x, \qquad (iii) \quad |x - 1| \ .$$

[3 marks]

3. Differentiate with respect to x

$$(i) \quad x^3 \ln x, \qquad (ii) \quad (x^4 + 1)^3, \qquad (iii) \quad \frac{x - 2}{x^2 + 4}.$$

[8 marks]

4. Given that

$$2x^3 + 3xy^2 + y^4 = 11$$

find $\frac{dy}{dx}$ in terms of x and y. [4 marks]

5. Find the following indefinite integrals

$$(i) \quad \int (4 - 3x)^5 \, dx, \qquad (ii) \quad \int \frac{x^3}{x^4 + 3} \, dx \ .$$

[6 marks]

6. Evaluate

$$(i) \quad \int_0^{\pi} x \sin 3x \, dx, \qquad (ii) \quad \int_1^2 \frac{dx}{(x + 3)^3} \ .$$

[8 marks]

7. Evaluate the sums

$$(i) \quad \sum_{k=0}^{10}(5+2k), \qquad (ii) \quad \sum_{k=1}^{6}\left(\frac{1}{2}\right)^k .$$

[4 marks]

8. Given that $z_1 = 1 - i$ and $z_2 = 3 - 2i$ determine, in the form $a + ib$,

$$(i) \quad 2z_1 + 3z_2, \qquad (ii) \quad z_1 z_2, \qquad (iii) \quad \frac{1}{z_1 z_2} ,$$

$$(iv) \quad z_1/z_2.)$$

Find also Arg z_1 .

[6 marks]

9. Find all the first and second partial derivatives of

$$f(x, y) = x^2 \cos(xy) .$$

[5 marks]

10. Find the Maclaurin series expansion for $\cosh 2x$ up to and including the term in x^4. [4 marks]

11. The letters $ACIISSSTTT$ are written on 10 otherwise identical cards. The cards are chosen randomly one at a time and placed in order. What is the probability that they spell $STATISTICS$? [4 marks]

SECTION B

12.

(a) Find constants A and B such that

$$\frac{x}{(x+1)(x+3)} = \frac{A}{x+1} + \frac{B}{x+3}.$$

Hence evaluate

$$\int_1^2 \frac{x\,dx}{(x+1)(x+3)}.$$

[6 marks]

(b) Find the indefinite integrals

$$(i) \quad \int \frac{(x+2)^2}{x^2+4} dx, \qquad (ii) \quad \int x^2 \cos x \, dx \ .$$

[9 marks]

13.

(a) Show that if $u(r,\theta) = r^3 \cos 3\theta$, then

$$u_{rr} + \frac{1}{r} u_r + \frac{1}{r^2} u_{\theta\theta} = 0 \ .$$

[6 marks]

(b) Evaluate

$$\int\int_A (1+2x)\, dx dy$$

where A is the region of the xy-plane bounded by the lines $x = 1, y = 1$ and $x + 2y = 5$.

[9 marks]

14.

(a) Express $(1+i)^7$ in the form $a+ib$ where a and b are real. [4 marks]

(b) Find all the roots of the equation

$$z^3 = 8i$$

in the form $a+ib$ where a and b are real. [6 marks]

(c) Use de Moivre's theorem to show that

$$\cos(3\theta) = 4\cos^3\theta - 3\cos\theta\ .$$

[5 marks]

15.

(a) Given that

$$y = f(x) = \frac{1}{x(x-2)}$$

find $f'(x)$ and $f''(x)$ *either* by first expanding $f(x)$ in partial fractions and then differentiating *or* by differentiating directly.

Show that the graph of $y = f(x)$ has one local maximum and determine its coordinates. Sketch the graph of $y = f(x)$ indicating any asymptotes.

[11 marks]

(b) Determine

$$\lim_{x\to 0}\frac{1-\cos 2x}{x^2}.$$

[4 marks]

16.

(a) A gardening society has 14 members of which 8 are men and 6 are women. Determine the number of ways in which a committee of 4 can be chosen. Also find the number of ways that the committee can be chosen if it includes at least one woman. [7 marks]

(b) A watch manufacturer finds that 4% of the watches produced are defective. These defects occur randomly in the manufacturing process.

Determine the probability that in a sample of 6 watches

(i) exactly 1 watch is defective;

(ii) fewer than 3 watches are defective.

[8 marks]