1. Give the names of the following (lower case) Greek letters: β, η. Write the lower case Greek letters *epsilon* and *omega*. [8 marks]

2. Say whether or not each of the following is a mathematical statement. For each that is, state whether it is true, false, or has free variables: if it has free variables, identify them.

a) $n^n e^{-n}\sqrt{2\pi n}$.

b) For all real numbers x and y, if $xy = 0$ then $x = 0$ or $y = 0$.

c) $\exists x \in \mathbb{R},\ f(x) > 0$ and $g(x) < 0$.

[10 marks]

3. For each of the following sets S, give a function $f(n)$ such that

$$S = \{f(n) \mid n \in \mathbb{N}\}.$$

(Note that 0 is considered to be a natural number.)

a) $S = \{1, 3, 5, 7, 9, \ldots\}$.

b) $S = \{2, 4, 8, 16, 32, 64, \ldots\}$.

c) $S = \mathbb{Z}$. [12 marks]

4. Negate each of the following statements:

a) $|f(x) - f(y)| < \epsilon$.

b) $x > 0$ or $f(x) < 0$.

c) If $|x| < \frac{1}{10}$ then $|f(x)| < \frac{1}{100}$.

d) $\forall x \in \mathbb{R},\ \exists y \in \mathbb{R},\ f(y) = x$. [12 marks]

5.

Definition: Let S be a subset of $\mathbb{Z}$. Then S is *closed under addition* if for all $m, n \in S$, $m + n \in S$.

Working directly from this definition, determine whether or not the following subsets S of $\mathbb{Z}$ are closed under addition. You should justify your answers.

a) $S = \{3k + 2 \mid k \in \mathbb{Z}\}$.

b) $S = \{k \in \mathbb{Z} \mid k < -10 \text{ or } k > 10\}$.

c) $S = \{k \in \mathbb{Z} \mid k > 10\}$.

d) $S = \{2^k \mid k \in \mathbb{N}\}$. [14 marks]

6. Consider the following theorem. You are not expected to understand what it means.

Theorem *Let X be a T_2-space. If X is first countable and X is countably compact, then X is T_3.*

Identify the context, hypothesis, and conclusion of the theorem. State its contrapositive.

What, if anything, does the theorem tell you about a T_2-space X which is:

a) Not countably compact?

b) Countably compact but not T_3?

c) Neither countably compact nor T_3?

d) First countable and countably compact? [14 marks]

7. Write proofs of the following statements. In parts a) and b), you should work from the definition:

Definition: Let $m, n \in \mathbb{Z}$. Then m *divides* n, written $m|n$, if there exists an integer k such that $n = km$.

a) Let $a, b, c \in \mathbb{Z}$. If $a|b$ then $a|bc$.

b) Let $a, b, c \in \mathbb{Z}$. If $a|b$ and $b|c$ then $a|c$.

c) Let $x, y \in \mathbb{R}$. If $x \neq y$ then $(x+y)^2 > 4xy$. [15 marks]

8. Determine whether each of the following statements is true or false. Justify your answers briefly.

a) $\forall x \in \mathbb{R},\ \sin x < 2$.

b) $\exists x \in \mathbb{R},\ x^2 + 1 = 1/2$.

c) $\forall x \in \mathbb{R},\ \exists y \in \mathbb{R},\ x < y - 1$.

d) $\forall x \in \mathbb{R},\ \exists y \in \mathbb{R},\ e^y = x$.

e) $\exists x \in \mathbb{R},\ \forall y \in \mathbb{R},\ x + y = 0$. [15 marks]