**1.** Give the names of the following (lower case) Greek letters: $\lambda$, $\omega$. Write the lower case Greek letters *delta* and *mu*. [8 marks]

**2.** For each of the following sets $S$, give a function $f(n)$ such that

$$S = \{f(n) \mid n \in \mathbb{N}\}.$$

(Note that 0 is considered to be a natural number.)

a) $S = \{5, 7, 9, 11, 13, 15, 17, \ldots\}$.

b) $S = \{10, 100, 1000, 10000, \ldots\}$.

c) $S = \{\ldots, -6, -4, -2, 0, 2, 4, 6, \ldots\}$. [12 marks]

**3.** Negate each of the following statements:

a) $x^2 < 1$.

b) $x < 0$ or $x \geq 1$.

c) If $x < y$ then $f(x) < f(y)$.

d) $\exists M \in \mathbb{N}, \forall x \in \mathbb{R}, f(x) \leq M$. [12 marks]

**4.**

*Definition:* Let $f(x)$ be a (real-valued) function. Then $f(x)$ is *injective* if for all $x, y \in \mathbb{R}$,

$$f(x) = f(y) \implies x = y.$$

Working directly from this definition, determine whether or not the following functions are injective. You should justify your answers.

a) $f(x) = 1 - 3x$.

b) $f(x) = \sin x$. [10 marks]

**5.**

*Definition:* Let $S$ be a subset of $\mathbb{Z}$. Then $S$ is *closed under addition* if for all $m, n \in S$, $m + n \in S$.

Working directly from this definition, determine whether or not the following subsets $S$ of $\mathbb{Z}$ are closed under addition. You should justify your answers.

a) $S = \{4k \mid k \in \mathbb{Z}\}$.

b) $S = \{k \in \mathbb{Z} \mid k \leq 100\}$.

c) $S = \{k \in \mathbb{Z} \mid k \geq 100\}$. [14 marks]

**6.** Consider the following theorem. You are not expected to understand what it means.

**Theorem** *Let $X$ be a metric space. If $X$ is complete and $X$ is totally bounded, then $X$ is compact.*

Identify the context, hypothesis, and conclusion of the theorem. State its contrapositive.

What, if anything, does the theorem tell you about a metric space $X$ which is:

a) Compact?

b) Not Compact?

c) Complete but not compact?

d) Complete but not totally bounded? [14 marks]

**7.** Write proofs of the following statements. You should work from the definition:

*Definition:* Let $m, n \in \mathbb{Z}$. Then $m$ *divides* $n$, written $m|n$, if there exists an integer $k$ such that $n = km$.

a) Let $m, n \in \mathbb{Z}$. If $3|m$ and $3|n$ then $3|(m+n)$.

b) Let $a, b, c \in \mathbb{Z}$. If $a|b$ and $b|c$ then $a|c$.

c) Let $m, n \in \mathbb{Z}$. If 3 divides $m$ and 3 doesn't divide $m+n$, then 3 doesn't divide $n$. (*Hint:* you can assume the result of part a).) [15 marks]

**8.** Determine whether each of the following statements is true or false. Justify your answers briefly.

a) $\exists n \in \mathbb{Z},\ n^2 = 3$.

b) $\forall n \in \mathbb{Z},\ n^2 > 0$.

c) $\forall x \in \mathbb{R},\ \exists y \in \mathbb{R},\ 2y = x$.

d) $\forall x \in \mathbb{R},\ \exists y \in \mathbb{R},\ \sin y = x$.

e) $\exists x \in \mathbb{R},\ \forall y \in \mathbb{R},\ -1 < x - y < 1$. [15 marks]