**1.** Give the names of the following (lower case) Greek letters: $\sigma$, $\chi$. Write the lower case Greek letters *psi* and *beta*. [8 marks]

**2.** State whether or not each of the following numbers:

$$\text{i)} \ -1; \qquad \text{ii)}\ 0; \qquad \text{iii)}\ 9.5; \qquad \text{iv)}\ 6;$$

is an element of each of the following sets. For each possible combination of one of i) – iv) with one of a) – c), you should either state explicitly that the given number is an element of the given set, or state explicitly that it isn't.

a) $\mathbb{Z}$.

b) $\{x \in \mathbb{R} \,|\, x > -1 \text{ and } x^2 < 20\}$.

c) $\{\frac{1}{2}n^2 - 3 \,|\, n \in \mathbb{Z}\}$. [12 marks]

**3.** Negate each of the following statements:

a) $a > 1$ or $a < -2$.

b) $x \leq y \leq z$.

c) If $x > y$ then $f(x) > f(y)$.

d) $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, x > y$ and $f(x) \leq f(y)$. [12 marks]

**4.**

*Definition:* Let $f(x)$ be a (real-valued) function. Then $f(x)$ is *injective* if for all $x, y \in \mathbb{R}$,

$$f(x) = f(y) \implies x = y.$$

a) Write down what it means for $f$ *not* to be injective.

b) Determine whether or not the function $f(x) = 8x+12$ is injective. You should justify your answer carefully, working directly from the definition.

c) Show that the function $f(x) = x^3 - x$ is not injective. Again justify your answer from the definition.

[10 marks]

**5.** Write down carefully the meaning of the statement that $m|n$ ('$m$ divides $n$'), where $m$ and $n$ are integers.

*Definition:* Let $R$ be a relation on a set $X$. Then $R$ is an *equivalence relation* if for all $x, y, z \in X$ the following three conditions hold:

i) $x\,R\,x$.

ii) If $x\,R\,y$ then $y\,R\,x$.

iii) If $x\,R\,y$ and $y\,R\,z$ then $x\,R\,z$.

Determine whether or not the following relations $R$ on the given sets $X$ are equivalence relations. You should justify your answers carefully, working directly from the definitions.

a) $X = \mathbb{R}$, $x\,R\,y$ if $x + y > 0$.

b) $X = \mathbb{Z}$, $x\,R\,y$ if $4|(x - y)$.

c) $X = \{2, 3, 5, 7, 11\}$, $x\,R\,y$ if $x|y$. [14 marks]

**6.**

a) Prove the following proposition:

Let $m$ and $n$ be integers. If $m$ and $n$ are odd then $3m + 5n$ is even.

State the converse of this proposition and also state, with a reason, whether the converse is true.

b) Let $P$ be the proposition:

Let $m, n \in \mathbb{Z}$. If $m + n$ is odd then $m$ is odd or $n$ is odd.

State and prove the contrapositive of $P$.

c) Prove the following proposition:

Let $a, b \in \mathbb{R}$. If $a \neq b$ then $(a + b)^2 > 4ab$. [15 marks]

**7.** Consider the following theorem. You are not expected to understand what it means.

**Theorem** *Let $F, G$ be unfoldings. If $F$ is versal and $F$ is induced from $G$ then $G$ is versal.*

Identify the context, hypothesis and conclusion of this theorem. State its contrapositive.

What, if anything, can you deduce from this theorem, given unfoldings $F, G$ for which

a) $F$ and $G$ are both versal?

b) $F$ is versal and $G$ is not versal?

c) $F$ is induced from $G$ and $G$ is versal?

d) $F$ is induced from $G$ and $G$ is not versal? [14 marks]

**8.** Determine whether each of the following statements is true or false. Justify your answers briefly. Recall that $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$.
In part e), $\mathbb{R}^+ = \{x \in \mathbb{R} \,|\, x > 0\}$.

a) $\forall n \in \mathbb{Z},\ n^2 + 1 > 0.$

b) $\exists n \in \mathbb{N},\ n^2 = 3.$

c) $\forall m \in \mathbb{N},\ \exists n \in \mathbb{N},\ m + n = 10.$

d) $\exists m \in \mathbb{N},\ \forall n \in \mathbb{N},\ m + n = n.$

e) $\forall y \in \mathbb{R}^+,\ \exists x \in \mathbb{R}^+,\ x < y^2.$ [15 marks]