1. Give the names of the following (lower case) Greek letters: γ, ψ. Write the lower case Greek letters *lambda* and *rho*. [8 marks]

2. Say whether or not each of the following is a mathematical statement. For each that is, state whether it is true, false, or has free variables: if it has free variables, identify them.

a) $x < y$.

b) $x \leq x^2$ for all real numbers x.

c) $\forall x \in \mathbb{R},\ f(x) = 0 \implies x > 2$.

[10 marks]

3. For each of the following sets S, give a function $f(n)$ such that

$$S = \{f(n) \mid n \in \mathbb{N}\}.$$

(Note that 0 is considered to be a natural number.)

a) $S = \{10, 20, 30, 40, 50, 60, \ldots\}$.

b) $S = \{\ldots, -3, -2, -1, 0, 1, 2, 3, 4\}$.

c) $S = \{\ldots, -5, -3, -1, 1, 3, 5, \ldots\}$. [12 marks]

4. Negate each of the following statements:

a) $m > 5$.

b) $x > 1$ or $x < -1$.

c) If $x < y$ then $f(x) < f(y)$.

d) $\forall K \in \mathbb{N}, \exists x \in \mathbb{R}, |f(x)| > K$. [12 marks]

5.

Definition: Let R be a relation on a set X. Then R is an *equivalence relation* if for all $x, y, z \in X$ the following three conditions hold:

i) $x\,R\,x$.

ii) If $x\,R\,y$ then $y\,R\,x$.

iii) If $x\,R\,y$ and $y\,R\,z$ then $x\,R\,z$.

Definition: Let $m, n \in \mathbb{Z}$. Then m *divides* n, written $m|n$, if there exists an integer k such that $n = km$.

Definition: Let $n \in \mathbb{Z}$. Then n is *prime* if $n \geq 2$, and there is no integer k with $1 < k < n$ such that $k|n$.

Working directly from these definitions, determine whether or not the following relations R on the given sets X are equivalence relations. You should justify your answers.

a) $X = \mathbb{Z}$, $x\,R\,y$ if $3|(x-y)$.

b) $X = \mathbb{R}$, $x\,R\,y$ if $x + y$ is an integer.

c) $X = \{n \in \mathbb{N} \,|\, n \geq 2\}$, $x\,R\,y$ if there is a prime number p which divides both x and y. [14 marks]

6. Consider the following theorem. You are not expected to understand what it means.

Theorem *Let X be a scheme. If X is reduced and X is irreducible, then X is integral.*

Identify the context, hypothesis, and conclusion of the theorem. State its contrapositive.

What, if anything, does the theorem tell you about a scheme X which is:

a) Not reduced?

b) Not integral?

c) Neither reduced nor irreducible?

d) Reduced and not integral? [14 marks]

7. Write proofs of the following statements. In part a), you should work from the definition:

Definition: Let $m, n \in \mathbb{Z}$. Then m *divides* n, written $m|n$, if there exists an integer k such that $n = km$.

a) Let $a, b, c \in \mathbb{Z}$. If $a|b$ then $a|(b + ac)$.

b) Let $x \in \mathbb{R}$. If $x^2 \geq -x$ then $x \geq 0$ or $x \leq -1$. (*Hint:* you can assume that if both sides of an inequality are divided by a negative number, then the inequality is reversed.)

c) There do not exist integers m and n with $3m - 6n = 2$. [15 marks]

8. Determine whether each of the following statements is true or false. Justify your answers briefly.

a) $\forall n \in \mathbb{N},\ 2n > 4$.

b) $\exists x \in \mathbb{R},\ \sin x < -0.5$.

c) $\forall x \in \mathbb{R}, \exists y \in \mathbb{R},\ y^2 = x$.

d) $\forall x \in \mathbb{R}, \exists y \in \mathbb{R},\ 3y + 2 = x$.

e) $\exists x \in \mathbb{R}, \forall y \in \mathbb{R},\ xy = 0$. [15 marks]