1. Give the names of the following (lower case) Greek letters: ϵ, ρ. Write the lower case Greek letters *beta* and *theta*. [8 marks]

2. For each of the following sets S, give a function $f(n)$ such that

$$S = \{f(n) \mid n \in \mathbb{N}\}.$$

(Note that 0 is considered to be a natural number.)

a) $S = \{1, 4, 7, 10, 13, 16, \ldots\}$.

b) $S = \{1, 2, 4, 8, 16, 32, \ldots\}$.

c) $S = \mathbb{Z}$. [12 marks]

3. Negate each of the following statements:

a) $x < 0$.

b) $-1 \leq y \leq 1$.

c) If $x < 0$ then $f(x) < 0$.

d) $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, f(y) = x$. [12 marks]

4.

Definition: Let $f(x)$ be a (real-valued) function. Then $f(x)$ is *injective* if for all $x, y \in \mathbb{R}$,

$$f(x) = f(y) \implies x = y.$$

Working directly from this definition, determine whether or not the following functions are injective. You should justify your answers.

a) $f(x) = 2x - 3$.

b) $f(x) = x^2$. [10 marks]

5.

Definition: Let R be a relation on a set X. Then R is an *equivalence relation* if for all $x, y, z \in X$ the following three conditions hold:

i) $x\,R\,x$.

ii) If $x\,R\,y$ then $y\,R\,x$.

iii) If $x\,R\,y$ and $y\,R\,z$ then $x\,R\,z$.

Working directly from this definition, determine whether or not the following relations R on the given sets X are equivalence relations. You should justify your answers.

a) $X = \mathbb{Z}$, $x\,R\,y$ if $x + y$ is even.

b) $X = \mathbb{R}$, $x\,R\,y$ if $x + 1 \geq y$.

c) $X = \mathbb{R}$, $x\,R\,y$ if $\sin x = \sin y$. [14 marks]

6. Consider the following theorem:

Theorem *Let p and a be positive integers. If p is prime and a is not divisible by p, then $a^{p-1} - 1$ is divisible by p.*

Identify the context, hypothesis, and conclusion of the theorem. State its contrapositive.

What, if anything, does the theorem tell you about positive integers a and p when

a) $a^{p-1} - 1$ is divisible by p?

b) $a^{p-1} - 1$ is not divisible by p?

c) Neither a nor $a^{p-1} - 1$ is divisible by p?

d) p is prime and a is divisible by p? [14 marks]

7. Write proofs of the following statements. In part a), you should work from the definition:

Definition: Let $n \in \mathbb{Z}$. Then n is *even* if there exists an integer k such that $n = 2k$.

a) Let $m, n \in \mathbb{Z}$. If m is even and n is even then $m + n$ is even.

b) Let $a, b \in \mathbb{R}$. If $a \neq b$ then $(a + b)^2 > 4ab$.

c) There do not exist integers m and n with $7m - 21n = 5$. [15 marks]

8. Determine whether each of the following statements is true or false. Justify your answers briefly.

a) $\exists n \in \mathbb{N},\ n^2 < 5.$

b) $\forall x \in \mathbb{R},\ x^2 < 5.$

c) $\forall x \in \mathbb{R}, \exists n \in \mathbb{Z},\ n < x.$

d) $\exists n \in \mathbb{Z}, \forall x \in \mathbb{R},\ n < x.$

e) $\exists x \in \mathbb{R}, \forall n \in \mathbb{N},\ x < n.$ [15 marks]