1. Give the names of the following (lower case) Greek letters: η, σ. Write the lower case Greek letters *gamma* and *psi*. [8 marks]

2. State whether or not each of the following numbers:

$$\text{i)} \ -1; \qquad \text{ii)} \ 0.5; \qquad \text{iii)} \ 2; \qquad \text{iv)} \ 5;$$

is an element of each of the following sets. For each possible combination of one of i) – iv) with one of a) – c), you should either state explicitly that the given number is an element of the given set, or state explicitly that it isn't.

a) $\mathbb{Z}$.

b) $\{x \in \mathbb{R} \,|\, x^2 < 5\}$.

c) $\{n^2 + 1 \,|\, n \in \mathbb{Z}\}$. [12 marks]

3. Negate each of the following statements:

a) $x^2 + y^2 < 1$.

b) $a = 0$ or $a > 1$.

c) If $x \leq y$ then $f(x) \geq f(y)$.

d) $\forall \epsilon > 0, \exists x \in \mathbb{R}, |f(x)| < \epsilon$. [12 marks]

4.

Definition: Let $f(x)$ be a (real-valued) function. Then $f(x)$ is *injective* if for all $x, y \in \mathbb{R}$,

$$f(x) = f(y) \implies x = y.$$

Determine whether or not the following functions are injective. You should justify your answers carefully, working directly from the definition.

a) $f(x) = (x+1)^2$.

b) $f(x) = 4 - x$.

[10 marks]

5.

Definition: Let R be a relation on $\mathbb{R}$. Then R is an *equivalence relation* if for all $x, y, z \in \mathbb{R}$ the following three conditions hold:

i) $x\,R\,x$.

ii) If $x\,R\,y$ then $y\,R\,x$.

iii) If $x\,R\,y$ and $y\,R\,z$ then $x\,R\,z$.

Determine whether or not the following relations R on $\mathbb{R}$ are equivalence relations. You should justify your answers carefully, working directly from the definitions.

a) $x\,R\,y$ if $e^x = e^y$.

b) $x\,R\,y$ if $x - 1 < y < x + 1$.

c) $x\,R\,y$ if $x - y \in \mathbb{Q}$. [14 marks]

6. Consider the following theorem. You are not expected to understand what it means.

Theorem *Let X be a T_2-space. If X is first countable and X is countably compact, then X is T_3.*

Identify the context, hypothesis, and conclusion of the theorem. State its contrapositive.

What, if anything, does the theorem tell you about a T_2-space X which is:

a) Not first countable?

b) First countable and not T_3?

c) Neither first countable nor T_3?

d) T_3 and countably compact? [14 marks]

7. Write proofs of the following statements. In part a), you should work from the definition:

Definition: Let $n \in \mathbb{Z}$. Then n is *even* if there exists an integer k such that $n = 2k$.

a) Let $m, n \in \mathbb{Z}$. If m is even and n is even then $m - n$ is even.

b) Let $a, b, c \in \mathbb{R}$. If $ab = ac$ then $a = 0$ or $b = c$.

c) There do not exist integers m and n with $2m + 4n = 1$. [15 marks]

8. Determine whether each of the following statements is true or false. Justify your answers briefly. In part e), $\mathbb{R}^+ = \{x \in \mathbb{R} \mid x > 0\}$.

a) $\exists n \in \mathbb{Z},\ n^2 > 0$.

b) $\forall n \in \mathbb{Z},\ n^2 > 0$.

c) $\forall x \in \mathbb{R},\ \exists y \in \mathbb{R},\ x + y = 0$.

d) $\exists x \in \mathbb{R},\ \forall y \in \mathbb{R},\ x + y = 0$.

e) $\forall \epsilon \in \mathbb{R}^+,\ \exists x \in \mathbb{R}^+,\ x^2 < \epsilon$. [15 marks]