Answer all of section A and THREE questions from section B. The marks shown against questions, or parts of questions, indicate their relative weights. Section A carries 55% of the total marks.

SECTION A

1. Write down the natural domain and the range of each of the following functions and sketch their graphs:

(i) $y = |x+3|$ (ii) $y = 1/(3x-2)^2$. (4 marks)

2. If

$$f(x) = \frac{1+2x}{1+3x},$$

find the rule of the inverse function $f^{-1}(x)$. Verify that

$$f(f^{-1}(x)) = x.$$ (5 marks)

3. State, giving reasons, whether or not the following functions are odd, even or neither:

(i) $f(t) = \dfrac{t^3 - 2t}{t^7 + t^5 + 2t^3 + t}$ (ii) $g(x) = \dfrac{\tan x}{x^2 + \cos x}$. (4 marks)

4. Find the following limits

(i) $\lim\limits_{p \to 3} \dfrac{p^3 - 3p^2 + 5}{p+3}$ (ii) $\lim\limits_{x \to -3} \dfrac{x^2 + 8x + 15}{x^2 + 5x + 6}$ (iii) $\lim\limits_{x \to 1} \dfrac{3x^2 - 3}{\ln x}$. (6 marks)

5. Write down the product rule for differentiation. Prove this rule by differentiation from first principles. (4 marks)

6. Differentiate the following identifying any rules of differentiation that you use

(i) $y = t^3 \sec(5t)$ (ii) $y = \dfrac{2+x}{1+x^2}$ (iii) $y = \exp(\cos(3x))$. (6 marks)

7. Use implicit differentiation to find expressions for dy/dx and d^2y/dx^2 in terms of x and y given that

$$xy + y^2 = 4x.$$ (6 marks)

8. Write down the definition of $\sinh x$ in terms of e^x and e^{-x}. Hence show that

$$\sinh^{-1}(x) = \ln\left(x + \sqrt{x^2+1}\right).$$ (5 marks)

9. Find the following indefinite integrals

(i) $\int \tan^4 x \sec^4 x \, dx$ (ii) $\int t^2 e^t \, dt$. (6 marks)

10. Find the arc length of the curve $y = x^{3/2}$ from $x = 0$ to $x = 4/9$. (5 marks)

11. Use the integral test to determine whether or not the following infinite series converges

$$\sum_1^{\infty} \frac{1}{n^5}.$$ (4 marks)

SECTION B

12. (i) Given that $x = a$ satisfies the equation $f(x) = x$, verify that $x = a$ also satisfies the equation $f(f(x)) = x$.

(ii) The function f is defined by

$$f(x) = x(2x-3).$$

Find all solutions of the equation

$$f(x) = x.$$

Verify that $f(f(x)) = 8x^4 - 24x^3 + 12x^2 + 9x$ and use the result of (i) to find <u>all</u> the solutions of the equation $f(f(x)) = x$.

(iii) Apply the Newton-Raphson method with initial value $x_0 = -0.5$ directly to $f(f(x)) - x = 0$. Hence find a solution of this equation correct to 3 decimal places. Compare your answer to that obtained in (ii) (15 marks)

13. Let

$$f(x)=\frac{x^2+4x-1}{x-1}.$$

Find constants *A*, *B*, and *C* such that

$$f(x)=Ax+B+\frac{C}{x-1}.$$

Find intervals of *x* on which the function is (i) increasing, (ii) decreasing, (iii) concave up and (iv) concave down. Locate any zeros, asymptotes, extrema and inflection points. Classify the extrema. Sketch the graph. (15 marks)

14. Peter is at a point A on the side of a long straight stretch of still water of width 0.1 miles. He enters the water directly and can swim at 2mph. He wishes to reach a point B on the opposite bank. If he swam directly across, he would then have to walk a distance of 0.5 miles to reach B. Suppose that he can walk/jog at 6mph. To which point on the opposite bank should he swim to get to his destination as soon as possible? How long will it take him to get to B? Give your answer correct to 3 decimal places. (15 marks)

15. Obtain approximate values for the integral

$$\int_1^2 \frac{1}{x+1}dx$$

using (i) the trapezoidal rule and (ii) Simpson's rule with the interval [1,2] subdivided into ten equal parts in each case. Give your answers correct to five decimal places. Compare your answers with the exact result and comment very briefly on your findings. (15 marks)

16. Let

$$I_n=\int \sin^n\theta d\theta.$$

Show by writing $\sin^n\theta=\sin^{n-1}\theta\sin\theta$ and integrating by parts, that

$$I_n=-(1/n)\sin^{n-1}\theta\cos\theta+((n-1)/n)(I_{n-2}).$$

Hence show that if

$$K_n=\int_0^{\pi/2}\sin^n\theta d\theta,$$

then $K_7=16/35$ and $K_8=35\pi/256$. Find general expressions for K_{2n} and K_{2n+1}. (15 marks)