Answer all of section A and THREE questions from section B. The marks shown against questions, or parts of questions, indicate their relative weights. Section A carries 55% of the total marks.

SECTION A

1. Find the general solution of

$$\cos\theta = 1/2 . \qquad \text{(4 marks)}$$

2. If

$$f(x) = \frac{1-3x}{1+x},$$

find the corresponding inverse function $f^{-1}(x)$. Verify that

$$f(f^{-1}(x)) = x. \qquad \text{(5 marks)}$$

3. State, giving reasons, whether or not the following functions are odd, even or neither:

(i) $f(x) = \dfrac{x^2 - 1}{x^4 + x^2 + 2}$ (ii) $g(x) = \dfrac{x^3 + \sin x}{\cos x}$. (4 marks)

4. Find the following limits

(i) $\lim\limits_{x\to 3} \dfrac{x^3 - 3x^2 + 5}{x+3}$ (ii) $\lim\limits_{x\to -2} \dfrac{x^2 + x - 2}{x^2 + 5x + 6}$ (iii) $\lim\limits_{x\to 1} \dfrac{\ln x}{x^2 - 1}$. (6 marks)

5. Find the equation of the tangent to the curve $y = \tan(\pi x/4)$ at the point (1,1).

(4 marks)

6. Differentiate the following identifying any rules of differentiation that you use

(i) $y = x^4 \sin(3x)$ (ii) $y = \dfrac{1-x^2}{1+x^2}$ (iii) $y = \exp(\tan(3x))$. (6 marks)

7. Use implicit differentiation to find expressions for dy/dx and d^2y/dx^2 in terms of x and y given that

$$xy + y^2 = 2x . \qquad \text{(6 marks)}$$

8. Find and classify the local extrema of the function

$$f(x) = x^3 - 3x^2 - 9x + 2\,.$$ (5 marks)

9. Evaluate the following integrals

(i) $\int \cos^8 x \sin x\, dx$ (ii) $\int_0^{\pi/4} x^2 \cos x\, dx$. (6 marks)

10. Find the arc length of the curve $y = x^{3/2}$ from $x = 0$ to $x = 5$. (5 marks)

11. Use D'Alembert's ratio test to determine whether or not the following infinite series converges

$$\sum_1^\infty \frac{100^n}{n!}\,.$$ (4 marks)

SECTION B

12. (i) Given that $x = a$ satisfies the equation $f(x) = x$, verify that $x = a$ also satisfies the equation $f(f(x)) = x$.

(ii) The function f is defined by

$$f(x) = x(x-3)\,.$$

Find solutions of the equation

$$f(x) = x\,.$$

Verify that $f(f(x)) = x^4 - 6x^3 + 6x^2 + 9x$ and use the result of (i) to find <u>all</u> the solutions of the equation $f(f(x)) = x$.

(iii) Apply the Newton-Raphson method with initial value $x_0 = 3$ directly to $f(f(x)) - x = 0$. Hence find a solution of this equation correct to 3 decimal places. Compare your answer to that obtained in (ii) (15 marks)

13. Let

$$f(x) = \frac{x^2 + x - 1}{x - 1}.$$

Find constants *A*, *B*, and *C* such that

$$f(x) = Ax + B + \frac{C}{x-1}.$$

Find intervals of *x* on which the function is (i) increasing, (ii) decreasing, (iii) concave up and (iv) concave down. Locate any zeros, asymptotes, extrema and inflection points. Classify the extrema. Sketch the graph. (15 marks)

14. Write down the definitions of $\cosh x$ and $\sinh x$ in terms of e^x and e^{-x}. Hence establish the following identity:

$$\sinh(x + y) = \sinh x \cosh y + \cosh x \sinh y .$$

A hyperbolic function is defined by $\operatorname{cosech} x = 1/\sinh x$. Show that its inverse function $\operatorname{cosech}^{-1} x$ can be represented by the formula:

$$\operatorname{cosech}^{-1} x = \ln\left(\frac{1}{x} + \sqrt{1 + \frac{1}{x^2}}\right) \quad (x \neq 0).$$

Use this formula to show that

$$\frac{d}{dx}\left(\operatorname{cosech}^{-1} x\right) = \frac{-1}{|x|\sqrt{1 + x^2}} \quad (x \neq 0).$$

(15 marks)

15. The function *f* is defined as follows:

$$f(x) = \begin{cases} (\sin x)/x & x \neq 0 \\ 1 & x = 0 \end{cases}.$$

Obtain approximate values for the integral

$$\int_0^1 f(x)dx$$

using (i) the trapezoidal rule and (ii) Simpson's rule with the interval $[0,1]$ subdivided into ten equal parts in each case. Give your answers correct to five decimal places. Explain very briefly which you expect to be the more accurate and why. (15 marks)

16. (i) Find the following indefinite integrals

(a) $\int e^{x} \sin 2x \; dx$ (b) $\int \frac{e^{-x} dx}{\sqrt{1-e^{-2x}}}$.

(ii) Find the volume of the solid generated by rotating the region between the curve $y = 2x - x^2$ and the x-axis through 2π radians about the x-axis. (15 marks)