PAPER CODE NO.
MATH 013

THE UNIVERSITY *of* LIVERPOOL

SEPTEMBER 2003 EXAMINATIONS

Bachelor of Engineering : Year 1
Bachelor of Science : Year 1

MATHEMATICAL METHODS

TIME ALLOWED : Three Hours

INSTRUCTIONS TO CANDIDATES

You may attempt all questions. All answers to Section A and the best THREE answers to Section B will be taken into account.
Numerical answers should be given correct to four places of decimals.

SECTION A

1. Determine the radian measure of the angle $\alpha = -600^0$, expressed as a rational multiple of π.
   Using the formula for $\sin(A-B)$, or otherwise, find the exact value for $\sin(\alpha)$, *without using tables or a calculator.*
   Hence determine all the angles θ, in the range $\left[-360^0, 360^0\right]$ satisfying $\sin(\theta) = \sin(\alpha)$.

   [7 marks]

2. Find all the solutions for θ in the range $\left[0, 360^0\right]$, which satisfy

   $$4\cos^2(\theta) + 6\sin^2(\theta) = 5.$$

   [6 marks]

3. Find (to 4 decimal places) the value of x which satisfies

   $$\log_e(2x) + \log_e(x^2) = 7.$$

   [5marks]

4. Use logarithms to solve the equation

   $$6^{3x-2} = 5^x.$$

   [5 marks]

5. Write down the first seven rows of Pascal's triangle. Hence or otherwise find the coefficient of x^8 in the expansion of

   $$(2x^2 - 3)^6.$$

   [6 marks]

6. Sketch the graph of the quadratic function $q(x) = 2x^2 - 3x - 5$. Determine the zeros of $q(x)$ and the position of its minimum.

[8 marks]

7. Express the rational function $f(x)$ in partial fractions, where

$$f(x) = \frac{2x^2 + 2x + 10}{(x+1)(x^2+9)}.$$

[8 marks]

8. Express the complex number

$$z = \frac{3 - 2\mathrm{i}}{4 + \mathrm{i}}$$

in the form $z = a + b\mathrm{i}$.
Calculate the modulus and argument of z. The argument should be expressed in radian measure. Hence, or otherwise, find the modulus and argument of z^2.

[10 marks]

SECTION B

9. Assuming the *Difference Formula* for the cosine function:

$$\cos(x-y)=\cos(x)\cos(y)+\sin(x)\sin(y),$$

show that $\cos(x-\pi/2)=\sin(x)$, for all *x*. [2marks]
Express $12\cos(x)+5\sin(x)$ in the form $A\cos(x-\phi)$, where the phase angle ϕ is acute and $A>0$. The angle should be expressed in radians.
Hence solve the equation

$$12\cos(x)+5\sin(x)=-\frac{13\sqrt{3}}{2},$$

where *x* is an obtuse angle. Comment on the case when the right hand side of this equation is replaced by $-13\sqrt{3}$.

[13 marks]

10. (i) On separate diagrams sketch the curves $y=\log_e(x)$ and $y=1-e^{-x}$ for $x>0$.

[4 marks]

(ii) Solve the following equations:

$$\log_2(x)=8, \qquad \log_y(625)=4.$$

[4 marks]

(iii) A swarm of locusts is plaguing a local farming community. The authorities decide to tackle the problem by spraying the fields with a powerful insecticide. The population of locusts $N(t)$, *t* days after the application of the insecticide is believed to satisfy

$$N=\alpha-20000\left(1-e^{(0.2-k)t}\right),$$

where α and *k* are constants. Initially it was estimated that there were 15000 locusts in the swarm, and after 5 days of insecticide spraying the population was estimated to have fallen by 60%. Use this information to calculate α and *k*. Estimate the number of days it takes to kill off the swarm entirely.

[7 marks]

11. (i) If α and β are the roots of the equation $5x^2 - 9x - 1 = 0$, write down the values of a) $\alpha\beta$, b) $\alpha + \beta$, c) $\alpha^2 + \beta^2$ and d) $(\alpha - \beta)^2$, *without determining the values of α and β* individually.

[6 marks]

(ii) Given the following cubic polynomial

$$p(x) = -4x^3 + 15x^2 - 8x - 3,$$

calculate the values of $p(-2), p(-1), p(0), p(1), p(2), p(3)$ and $p(4)$. Hence find all the roots $p(x) = 0$, and sketch the curve.

[9 marks]

12. (i) A complex number has modulus 1 and argument $2\pi/3$. Express each of the following complex numbers in the form $a + b\mathrm{i}$:

$$z,\ z^2,\ z^3,\ \frac{1}{z},$$

and plot them (separately) on the Argand diagram.

[10 marks]

(ii) If $(x + \mathrm{i}y)^3 = (a + \mathrm{i}b)$ show that $a^2 + b^2 = (x^2 + y^2)^3$.

[5 marks]