PAPER CODE NO.
MATH 013

THE UNIVERSITY
of LIVERPOOL

SEPTEMBER 2005 EXAMINATIONS

Bachelor of Engineering : Foundation Year
Bachelor of Science : Foundation Year

MATHEMATICAL METHODS

TIME ALLOWED : Three Hours

INSTRUCTIONS TO CANDIDATES

You may attempt all questions. All answers to Section A and the best THREE answers to Section B will be taken into account.
Numerical answers should be given correct to four places of decimals.

SECTION A

1. Determine the radian measure of the angle $\alpha = -150^0$, expressed as a rational multiple of π.
   The formula for $\cos(A+B)$ states that

   $$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B).$$

   Using this formula or otherwise, find the exact value for $\cos(\alpha)$, *without using tables or a calculator*. (Show all your working).
   Hence determine all the angles θ, in the range $[0^0, 720^0]$ satisfying $\cos(\theta) = \cos(\alpha)$. Your answers can be expressed in degrees or radians.

   [6 marks]

2. Find all the solutions for θ in the range $[0, 360^0]$, which satisfy

   $$\sin(2\theta) - \cos(\theta) = 0.$$

   [7 marks]

3. Find (to 4 decimal places) the value of x which satisfies

   $$\log_{10}(2x) + \log_{10}(x^2) = 4.$$

   [7 marks]

4. Use logarithms to solve the equation

   $$5^{x+3} = 3^{x+1}.$$

   [8 marks]

5. Write down the first six rows of Pascal's triangle. Hence or otherwise find the coefficient of x^3 in the expansion of

   $$(2x+1)^5.$$

   [6 marks]

6. Let $q(x)$ be the quadratic function $q(x) = 2x^2 + 3x - 5$. Determine the zeros of $q(x)$ and the position of its minimum. Hence sketch the graph of $q(x)$.

[7 marks]

7. Express the rational function $f(x)$ in partial fractions, where

$$f(x) = \frac{2x+1}{(2x-3)(x-4)}.$$

[5 marks]

8. Express the complex number

$$z = \frac{2-3\mathrm{i}}{4+3\mathrm{i}}$$

in the form $z = a + b\mathrm{i}$.
Determine numerically the modulus and argument of z. The argument should, preferably, be expressed in radian measure. Hence, or otherwise, find the modulus and argument of z^2.

[9 marks]

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SECTION B

9. Assuming the *Difference Formula* for the cosine function:

$$\cos(x-y)=\cos(x)\cos(y)+\sin(x)\sin(y),$$

show that $\cos(x-\pi)=-\cos(x)$, for all x. [2marks]
Express $5\cos(x)+12\sin(x)$ in the form $A\cos(x-\phi)$, where the phase angle ϕ is acute and $A>0$. The angle should be expressed in radians.
Hence solve the equation

$$5\cos(x)+12\sin(x)=\frac{13}{\sqrt{2}},$$

where $0\le x\le\pi$. Comment on the case when the right hand side of this equation is replaced by $13\sqrt{2}$.

[13 marks]

10. (i) On separate diagrams sketch the curves $y=-\log_e(x)$ for $x>0$ and $y=2e^x$ for all x.

[4 marks]

(ii) Solve the following equations:

$$\log_3(x)=-2, \qquad \log_y(343)=3.$$

[4 marks]

(iii) A chemical substance B can be converted into another substance by means of a chemical reaction. During the reaction the amount of B (in grams) left unconverted at time t is given by

$$B=Ce^{-kt},$$

where C and k are constants. Initially, before the reaction began, there was 10g of B, and after 10 sec only one fourth of the substance had been converted. Calculate both C and k. Find how long it will take for nine tenths of the substance to have undergone conversion.

[7 marks]

11. (i) If α and β are the roots of the equation $4x^2+4x-1=0$, write down the values of a) $\alpha\beta$, b) $\alpha+\beta$, c) $\alpha^2+\beta^2$ and d) $(\alpha-\beta)^2$, *without determining the values of* α *and* β individually.

[8 marks]

(ii) Given the following cubic polynomial

$$p(x)=4x^3+4x^2-29x-15,$$

calculate the values of $p(-4), p(-3), p(-2), p(-1), p(0), p(1)$ and $p(2)$. Hence find all the roots $p(x)=0$, and sketch the curve of $y=p(x)$.

[7 marks]

12. (i) A complex number z has modulus 1 and argument $3\pi/4$. Express each of the following complex numbers in the form $a+b\mathrm{i}$:

$$z,\ z^2,\ z^3,\ \frac{1}{z},$$

and plot them (separately) on the Argand diagram.

[10 marks]

(ii) If $z=2+3\mathrm{i}$ and $w=1-2\mathrm{i}$ are two complex numbers, calculate (in the form $a+b\mathrm{i}$) the following:

a) zw, b) $\dfrac{2z}{w}$.

[5 marks]