PAPER CODE NO.
MATH 013

THE UNIVERSITY
of LIVERPOOL

SEPTEMBER 2004 EXAMINATIONS

Bachelor of Engineering : Year 1
Bachelor of Science : Year 1

MATHEMATICAL METHODS

TIME ALLOWED : Three Hours

INSTRUCTIONS TO CANDIDATES

You may attempt all questions. All answers to Section A and the best THREE answers to Section B will be taken into account.
Numerical answers should be given correct to four places of decimals.

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SECTION A

1. Determine the radian measure of the angle $\alpha = -480^0$, expressed as a rational multiple of π.
   Using the formula for $\sin(A+B)$, or otherwise, find the exact value for $\sin(\alpha)$, *without using tables or a calculator.*
   Hence determine all the angles θ, in the range $[-360^0, 360^0]$ satisfying $\sin(\theta) = \sin(\alpha)$.

   [7 marks]

2. Find all the solutions for θ in the range $[0, 180^0]$, which satisfy

$$2\sec^2(\theta) + 3\tan(\theta) = 1.$$

   [6 marks]

3. Find (to 4 decimal places) the value of x which satisfies

$$\log_e(4x) + \log_e(x^3) = 9.$$

   [5marks]

4. Use logarithms to solve the equation

$$7^{6-x} = 4^x.$$

   [5 marks]

5. Write down the first seven rows of Pascal's triangle. Hence or otherwise find the coefficient of x^9 in the expansion of

$$(x^3 + 2)^6.$$

   [6 marks]

6. Sketch the graph of the quadratic function $q(x) = 2x^2 - 13x - 7$. Determine the zeros of $q(x)$ and the position of its minimum.

[8 marks]

7. Express the rational function $f(x)$ in partial fractions, where

$$f(x) = \frac{3x - 4}{(x+1)(x+6)}.$$

[8 marks]

8. Express the complex number

$$z = \frac{1 + 2\mathrm{i}}{8 + 2\mathrm{i}}$$

in the form $z = a + b\mathrm{i}$.
Calculate the modulus and argument of z. The argument should be expressed in radian measure. Hence, or otherwise, find the modulus and argument of z^2.

[10 marks]

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SECTION B

9. Assuming the *Difference Formula* for the cosine function:

$$\sin(x-y)=\sin(x)\cos(y)-\cos(x)\sin(y),$$

show that $\sin(x-\pi/2)=-\cos(x)$, for all x. [2marks]

Express $4\sin(x)-3\cos(x)$ in the form $A\sin(x-\phi)$, where the phase angle ϕ is acute and $A>0$. The angle should be expressed in radians.

Hence solve the equation

$$4\sin(x)-3\cos(x)=\frac{5\sqrt{3}}{2},$$

where $0\le x\le\pi$. Comment on the case when the right hand side of this equation is replaced by $5\sqrt{3}$.

[13 marks]

10. (i) On separate diagrams sketch the curves $y=\log_e(x)$ for $x>0$ and $y=e^{-x}$ for all x.

[4 marks]

(ii) Solve the following equations:

$$\log_2(x)=5, \qquad \log_y(243)=5.$$

[4 marks]

(iii) A thermometer is used to measure the temperature of a house. Inside the house the temperature is 20^0C. At time $t=0$ the thermometer is moved to the outside of the house, where the air temperature is only 5^0C. Three minutes later the reading of the thermometer has dropped to 10^0C. Assuming the temperature of the thermometer, *T*, drops according to Newton's Law of cooling, one can show that

$$T=5+Qe^{-kt}.$$

Use the above information to calculate the two constants *Q* and *k*. How long after being put outside does it take the thermometer to register 7^0C ?

[7 marks]

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11. (i) If α and β are the roots of the equation $-3x^2+5x-3=0$, write down the values of a) $\alpha\beta$, b) $\alpha+\beta$, c) $\alpha^2+\beta^2$ and d) $(\alpha-\beta)^2$, *without determining the values of* α *and* β individually.

[6 marks]

(ii) Given the following cubic polynomial

$$p(x)=2x^3+6x^2-4x-4,$$

calculate the values of $p(-4), p(-3), p(-2), p(-1), p(0), p(1)$ and $p(2)$. Hence find all the roots $p(x)=0$, and sketch the curve.

[9 marks]

12. (i) A complex number has modulus 1 and argument $5\pi/6$. Express each of the following complex numbers in the form $a+b\mathrm{i}$:

$$z,\ z^2,\ z^3,\ \frac{1}{z},$$

and plot them (separately) on the Argand diagram.

[10 marks]

(ii) If $(x+\mathrm{i}y)^2=a+\mathrm{i}b$ show that $x^2-y^2=a$, $2xy=b$. Hence evaluate $\sqrt{8+6\mathrm{i}}$.

[5 marks]