PAPER CODE NO.
**MATH 013**

THE UNIVERSITY
*of* LIVERPOOL

# SEPTEMBER 2006 EXAMINATIONS

Bachelor of Engineering : Foundation Year
Bachelor of Science : Foundation Year

**MATHEMATICAL METHODS**

TIME ALLOWED : **Three Hours**

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INSTRUCTIONS TO CANDIDATES

You may attempt all questions. All answers to Section A and the best THREE answers to Section B will be taken into account.
Numerical answers should be given correct to four places of decimals.

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SECTION A

**1.** Determine the degree measure of the angle $\alpha = 13\pi/6$ radians.
The formula for $\tan(A+B)$ states that

$$\tan(A+B) = \frac{\tan(A)+\tan(B)}{1-\tan(A)\tan(B)}.$$

Using this formula or otherwise find the exact value for $\tan(\alpha)$, *without using tables or a calculator*. (Show all your working.)
Hence determine all the angles $\theta$, in the range $[0, 2\pi]$ satisfying $\tan(\theta) = \tan(\alpha)$. Your answers should be expressed in radian measure.

[6 marks]

**2.** Sketch the graph of $y = \cos(x)$ in the range $-270^0 \le x \le 270^0$. Determine numerically the solutions of $\cos(x) = -1/4$ in the same range. You may express your answers in degrees or radians.

[9 marks]

**3.** Use logarithms to find the value of *x* which satisfies the following equation.

$$5^{x+1} = 7^{x-1}.$$

[7 marks]

**4.** You are given the values of $\log_{10}(12) = 1.079181$ and $\log_{10}(4) = 0.602060$, correct to six decimal places. Obtain the values of the following

$$\log_{10}(48), \qquad \log_{10}(3), \qquad \log_{10}(64),$$

*without using tables or a calculator*, correct to four decimal places. (Show all your working.)

[6 marks]

**5.** Write down the first five rows of Pascal's triangle. Hence or otherwise find the coefficient of $x^6$ in the expansion of

$$(x^2 - 2)^4.$$

[6 marks]

**6.** Let $q(x)$ be the quadratic function $q(x)=2x^2-7x+3$. Determine the zeros of $q(x)$ and the position and nature of its turning point. Hence sketch the graph of $q(x)$.

[7 marks]

**7.** Express the rational function $f(x)$ in partial fractions, where

$$f(x)=\frac{6}{(2x+1)(x+3)}.$$

[5 marks]

**8.** Express the complex number

$$z=\frac{3+4\mathrm{i}}{6+2\mathrm{i}}$$

in the form $z=a+b\mathrm{i}$ where $a$ and $b$ are real.
Determine numerically the modulus and argument of $z$. The argument should, preferably, be expressed in radian measure. Hence, or otherwise, find the modulus and argument of $z^2$.

[9 marks]

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SECTION B

**9.** Given the fact that $\sec^2(\theta) = 1 + \tan^2(\theta)$, find the values of the angle $\theta$ lying in the range $[0, \pi]$ radians, which satisfy the equation

$$3\sec^2(\theta) = 5 + \tan(\theta).$$

[7 marks]

Using the identity $\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$ express $5\sin(x) + 12\cos(x)$ in the form $C\sin(x+\alpha)$, where $C > 0$ and $\alpha \in [0, \pi/2]$ are constants. Hence solve the equation

$$5\sin(x) + 12\cos(x) = \frac{13}{2},$$

for $x$ in the range $[0, 2\pi]$ radians.

[8 marks]

**10.** (i) On separate diagrams sketch the curves $y = e^{-x}$ for real $x$, and $y = \log_e(x+1)$ for $x > -1$.

[4 marks]

(ii) Solve the following equations:

$$\log_9(x) = 2, \qquad \log_y(81) = 4 .$$

[4 marks]

(iii) A farmer estimated the number of badgers $N$ living on his land varied with time $t$ (where $t$ is measured in years) according to the following equation

$$N = 1 + \frac{A}{4e^{-kt} + 1},$$

where $A$ and $k$ are constants. Initially at $t = 0$ he counted 10 badgers living on his land. Calculate the value of $A$. After 3 years he believed this had risen to 31. Calculate $k$ and use its value to estimate how many badgers would be living on the land in five years time.

[7 marks]

**11.** (i) If $\alpha$ and $\beta$ are the roots of the equation $2x^2+3x+3=0$, find the values of a) $\alpha\beta$, b) $\alpha+\beta$, c) $\alpha^2+\beta^2$ and d) $(\alpha-\beta)^2$, *without determining the values of $\alpha$ and $\beta$* individually.

[8 marks]

(ii) Plot a table of the values of the following cubic polynomial

$$p(x) = 2x^3 - 4x^2 - \frac{11}{2}x - \frac{3}{2},$$

for $x = -2, -1, 0, 1, 2, 3$ and $4$. Sketch the curve of the polynomial, and find all the roots of $p(x) = 0$.

[7 marks]

**12.** (i) Find the modulus and argument of the complex number $z = \frac{\sqrt{3}}{2} - \frac{1}{2}\mathrm{i}$. Plot $z$ on the Argand diagram. Hence or otherwise plot both $z^2$ and $\frac{1}{z}$ on the same diagram.

[10 marks]

(ii) If $w$ is a complex number with modulus one and argument $\frac{\pi}{3}$ calculate both a) $w+z$ and b) $wz$, where $z = \frac{\sqrt{3}}{2} - \frac{1}{2}\mathrm{i}$ as above.

[5 marks]