SECTION A

1. Determine the radian measure α_0 of the angle with degree measure 390^o, expressed as a rational multiple of π.
Without using tables or a calculator, find the exact value of $\sin(\alpha_0)$.
Find all angles α with $\cos(\alpha) = \sin(\alpha_0)$.

[7 marks]

2. Sketch the graph of $y = \tan(x)$ in the range $-\pi \leq x \leq 2\pi$.
Determine all solutions of $\tan(x) = 0.9608$ in the same range.
Express your solutions in radian measure.

[8 marks]

3. Simplify the following surds without using a calculator :

$$((4 + 7^{\frac{1}{2}})(4 - 7^{\frac{1}{2}}))^{\frac{1}{2}}; \quad (\sqrt{100} + \sqrt{36})^{\frac{1}{4}};$$

$$a^{\frac{3}{5}} b^{-\frac{2}{5}} - (\frac{b}{a})^2, \quad \text{where} \quad b = a^{\frac{3}{2}}.$$

[7 marks]

4. Sketch separate graphs of the functions

$$y = e^x \quad \text{and} \quad y = \ln(x).$$

How are the shapes of these graphs related ?

[5 marks]

5. Find the coefficient of x^4 in the expansion of

$$2\left(3x^3 - 4x^{-2}\right)^3 + \left(1 + 3x^2\right)^4.$$

[7 marks]

6. Sketch the graph of the quadratic function $q(x) = 2x^2 - x - 1$.
Determine the zeros of $q(x)$ and the position of its minimum.

[6 marks]

7. Express the rational function $f(x)$ in partial fractions, where

$$f(x) = \frac{2x - 5}{(x+1)(x-2)}.$$

[7 marks]

8. Express the complex number

$$z = \frac{5 - 7i}{2 + 3i}$$

in the form $z = a + ib$.
Calculate the modulus and argument of z. Express the argument in radian measure.

[8 marks]

SECTION B

9. (i) By drawing an appropriate triangle, in each case, prove the following exact results :

(a) $\sin(\frac{\pi}{4}) = \cos(\frac{\pi}{4}) = \frac{1}{\sqrt{2}}$;

(b) $\sin(\frac{\pi}{3}) = \frac{\sqrt{3}}{2}, \cos(\frac{\pi}{3}) = \frac{1}{2}$;

and for acute angles x, the identities:

(c) $\sin(\frac{\pi}{2} - x) = \cos(x)$;

(d) $\sin^2(x) + \cos^2(x) = 1$.

(ii) Assuming the *Difference Formula* for the sine function :

$$\sin(x - y) = \sin(x)\cos(y) - \cos(x)\sin(y),$$

and noting that $\frac{1}{3} - \frac{1}{4} = \frac{1}{12}$, show that

$$\sin(\frac{\pi}{12}) = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

.

Obtain a similar expression for $\cos(\frac{\pi}{12})$.

[15 marks]

10. (i) What is meant by the *base* of a logarithm ?
Solve the equations :

$$\log_2(x) = 3; \;\; \log_y(2) = -2; \;\; z\log_7(49) = 4z - 10,$$

for x, y and z.

(ii) The temperature in a radiator, $\Theta(t)$, decreases according to the formula

$$\Theta(t) = 40e^{-kt},$$

where t is the time in hours, k is a heat dissipation constant, and temperature is measured in degrees Centigrade.
What is the temperature of the radiator at $t = 0$?
After 30 minutes, the temperature of the radiator is measured at 21^oC.
Determine the value of k and find how long it would take for the temperature to fall to 10^oC. [15 marks]

11. Sketch the graphs of the polynomials :

$$p(x) = 2x^2 - x - 6 \;\text{ and }\; q(x) = 2x^3 - 5x^2 + x + 2.$$

Find the roots of $p(x)$ and $q(x)$, and deduce that they share a common factor.
Hence express the rational function $p(x)/q(x)$ in partial fractions. [15 marks]

12. (i) A complex number z has modulus one and argument $\frac{\pi}{3}$.
Using the information given in Question 9(i), express each of the following complex numbers in the form $a + ib$:

$$z,\; z^2,\; z^3,\; z^4,\; \frac{1}{z},\; z - \frac{1}{z},$$

and plot them on an Argand diagram.

(ii) Sketch the graph of the quadratic $r(x) = x^2 - 2x + 2$.
Deduce that $r(x)$ must have complex roots. Find these roots and plot them on an Argand diagram. [15 marks]