SECTION A

**1.** Determine the radian measure $\alpha_0$ of the angle with degree measure $510^o$, expressed as a rational multiple of $\pi$.
*Without using tables or a calculator*, find the exact value of $\sin(\alpha_0)$.
Find all angles $\alpha$ with $\cos(\alpha) = \sin(\alpha_0)$. [8 marks]

**2.** Sketch the graph of $y = \tan(x)$ in the range $-2\pi \leq x \leq 2\pi$.
Determine numerically all solutions of $\tan(x) = 1.45$ in the same range. Your solutions should be expressed in radian measure. [8 marks]

**3.** You are given the values: $\ln(2) = 0.693147$ and $\ln(5) = 1.609438$, correct to 6 places of decimals. Obtain values of the following

$$\ln(4),\ \ln(10),\ \ln(2.5),\ \ln(0.4),$$

*without using tables or a calculator*, correct to 4 places of decimals.
You should show your numerical working. [7 marks]

**4.** Determine the set of values of $x$ for which

$$\frac{1}{8} \leq 2^x < 64.$$

[3 marks]

**5.** Construct the first six rows of Pascal's triangle. Hence, or otherwise, find the coefficient of $x^{-5}$ in the expansion of

$$\left(2x^3 - \frac{1}{x^2}\right)^5.$$

[8 marks]

**6.** Sketch the graph of the quadratic function $q(x) = x^2 - 7x + 12$.
Determine the zeros of $q(x)$, and the position of the minimum. [6 marks]

**7.** Express the rational function $f(x)$ in partial fractions, where

$$f(x) = \frac{3x - 5}{(x - 2)(x + 1)}.$$

[7 marks]

**8.** Express the complex number

$$z = \frac{3 + i}{4 - 3i}$$

in the form $z = a + ib$.
Determine numerically the modulus and argument of $z$. The argument should be given in radian measure. [8 marks]

SECTION B

**9.** State the Addition and Difference Formulae for the cosine function.
Use these formulae to show that

$$\cos^2(x) + \sin^2(x) = 1,$$

and

$$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1.$$

Deduce that

$$\cos(4x) = 8\cos^4(x) - 8\cos^2(x) + 1.$$

Verify that this identity is satisfied when $x = 0, \frac{\pi}{2}, \frac{\pi}{4}$ and $\frac{\pi}{3}$. [15 marks]

**10.**(i) Simplify the equation

$$3e^{2\ln(x-4)} + \ln(e^{-7x}) + 2 = 0$$

.

Solve the simplified equation for $x$.

(ii) Use logarithms to solve the equation $2^{5+y} = 7^{2y-1}$ numerically for $y$.

(iii) Sketch the graphs of the functions $e^x$ and $2e^{-x}$ on the same diagram, and determine their point of intersection. [15 marks]

**11.**(i) The quadratic $x^2 + 3x - 8$ has roots $\alpha, \beta$. Write down the values of $\alpha + \beta$ and $\alpha\beta$.

*Without calculating* $\alpha, \beta$, find the value of

$$\frac{1}{\alpha} + \frac{1}{\beta}.$$

Hence determine a quadratic whose roots are $\frac{1}{\alpha} - \alpha$ and $\frac{1}{\beta} - \beta$.

(ii) Sketch the graph of the cubic polynomial $p(x) = 2x^3 - 9x^2 + 7x + 6$. Find all the zeros of $p(x)$. [15 marks]

**12.**(i) The complex number $z_1$ has modulus one and argument $\frac{\pi}{6}$.
The complex number $z_2$ has modulus one and argument $\frac{\pi}{3}$.
Express each of the following complex numbers in the form $a + ib$ :

$$z_1,\ z_2,\ z_1 + z_2,\ z_1 z_2,\ \frac{z_1}{z_2},$$

and plot them on an Argand diagram.

(ii) Sketch the graph of the quadratic $q(x) = x^2 + 2x + 2$.
Deduce that $q(x)$ must have complex roots. Find these roots and plot them on an Argand diagram. [15 marks]