## SECTION A

**1.** Determine the radian measure $\alpha_0$ of the angle with degree measure $495^o$, expressed as a rational multiple of $\pi$.
*Without using tables or a calculator*, find the exact value of $\tan(\alpha_0)$.
Find all angles $\alpha$ with $\tan(\alpha) = \tan(\alpha_0)$. [8 marks]

**2.** Sketch the graph of $y = \sec(x)$ in the range $-2\pi \leq x \leq 2\pi$.
Determine numerically all solutions of $\sec(x) = 3$ in the same range. Your solutions should be expressed in radian measure. [8 marks]

**3.** Show that

$$2\ln(2+\sqrt{3}) + \ln(7-4\sqrt{3}) = 0,$$

*without using tables or a calculator.* [4 marks]

**4.** Determine the set of values of $x$ for which

$$-2 \leq \log_3(x) < 3.$$

[4 marks]

**5.** Construct the first seven rows of Pascal's triangle. Hence, or otherwise, find the coefficient of $x^3$ in the expansion of

$$\left(2x^2 - \frac{3}{x}\right)^6.$$

[7 marks]

**6.** Sketch the graph of the quadratic function $f(x) = 2x^2 + 3x - 2$.
Determine the zeros of $f(x)$, and the position of its minimum. [8 marks]

**7.** Express the rational function $f(x)$ in partial fractions, where

$$f(x) = \frac{5x - 4}{(x+1)(x-2)}.$$

[8 marks]

**8.** Express the complex number

$$z = \frac{8 + 7i}{3 - i}$$

in the form $z = a + ib$.
Determine numerically the modulus and argument of $z$. The argument should be given in radian measure. [8 marks]

## SECTION B

**9.** (i) State the Addition Formula for the cosine function.
Use this formula to show that

$$\cos(2\alpha) = 2\cos^2(\alpha) - 1.$$

*Without the use of tables or a calculator*, show that if an acute angle $\alpha$ satisfies $\cos(2\alpha) = \frac{1}{8}$, it also satisfies $\cos(\alpha) = \frac{3}{4}$.

(ii) Write $4\sin(x) - 3\cos(x)$ in the form $A\sin(x - \phi)$, where the phase angle $\phi$ is acute and $A > 0$. The angle $\phi$ should be expressed numerically in radians.
Hence solve the equation

$$4\sin(x) - 3\cos(x) = \frac{5}{\sqrt{2}},$$

where $x$ is an obtuse angle. [15 marks]

**10.** (i) Use logarithms to solve the equation $3^{x+2} = 5^{2x-1}$ numerically for $x$.

(ii) Solve the equation $9^{2-y} = 27^{2y-3}$ for $y$, expressing the solution as a simple fraction.

(iii) Sketch the graphs of the functions $2e^x$ and $e^{-x}$ on the same diagram.

Determine the common value of the functions at their point of intersection. You should not find it necessary to find the value of $x$ at the point of intersection. [15 marks]

**11.** (i) The quadratic $x^2 - 7x + 11$ has roots $\alpha, \beta$. Write down the values of $\alpha + \beta$ and $\alpha\beta$.

*Without calculating* $\alpha, \beta$, find the value of

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha}.$$

Hence determine a quadratic whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$.

(ii) Given that the cubic $x^3 + 2x^2 - 9x - 18$ has a small negative integer as a root, find that root. Hence find all the roots of the cubic.

Calculate the remainder when the cubic is divided by $x + 1$. [15 marks]

**12.** (i) A complex number $z$ has modulus one and argument $\frac{\pi}{4}$. Express each of the following complex numbers in the form $a + ib$ :

$$z,\ z^2,\ z^3,\ z^4,\ z^7,\ \frac{1}{z}, \sqrt{2}z^{2401},$$

and plot them on an Argand diagram.

(ii) Determine the (complex) roots of the quadratic $x^2 - 4x + \frac{25}{4}$. Verify that the two roots have the same modulus. Show also that the product of the two roots is the square of the common modulus.

What is the relationship between the arguments of the two roots? [15 marks]