PAPER CODE NO.
MATH 013

THE UNIVERSITY
of LIVERPOOL

JANUARY 2007 EXAMINATIONS

Bachelor of Engineering : Foundation Year
Bachelor of Science : Foundation Year

MATHEMATICAL METHODS

TIME ALLOWED : **Three Hours**

INSTRUCTIONS TO CANDIDATES

You may attempt all questions. All answers to Section A and the best THREE answers to Section B will be taken into account.
Numerical answers should be given correct to four places of decimals.

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SECTION A

1. If α represents the angle $11\pi/6$ measured in radians, what is the value of α measured in degrees?
The formula for $\cos(A-B)$ states that

$$\cos(A-B)=\cos(A)\cos(B)+\sin(A)\sin(B).$$

Using this formula or otherwise find the exact value for $\cos(\alpha)$, *without using tables or a calculator*. (Show all your working.)
Hence determine all the angles θ, in the range $[0, 2\pi]$ satisfying $\sin(\theta)=\cos(\alpha)$. Your answers can be expressed in degrees or radians.

[7 marks]

2. Sketch the graph of $y=\cos(2x)$ in the range $0^0 \le x \le 360^0$. Determine numerically the solutions of $\cos(2x)=0.8$ for x in the same range.

[9 marks]

3. Use logarithms to solve the equation $5^{x+1}=3^{x+2}$, giving x to 4 decimal places.

[6 marks]

4. You are given the values of $\log_{10}(45)=1.653212$ and $\log_{10}(3)=0.477121$, correct to six decimal places. Obtain the values of the following

$$\log_{10}(135), \qquad \log_{10}(15), \qquad \log_{10}(27),$$

without using tables or a calculator, correct to four decimal places. (Show all your working.)

[6 marks]

5. Write down the first six rows of Pascal's triangle. Hence or otherwise find the coefficient of x^3 in the expansion of

$$(2+3x)^5.$$

[6 marks]

6. Let $q(x)$ be the quadratic function $q(x) = 2x^2 - 7x - 4$. Determine the zeros of $q(x)$ and the position and nature of its turning point. Hence sketch the graph of $q(x)$.

[7 marks]

7. Express the rational function $f(x)$ in partial fractions, where

$$f(x) = \frac{3x}{(x-3)(x+8)}.$$

[5 marks]

8. Express the complex number

$$z = \frac{2 - \mathrm{i}}{3 - 2\mathrm{i}}$$

in the form $z = a + b\mathrm{i}$ where a and b are real.
Determine numerically the modulus and argument of z. The argument should, preferably, be expressed in radian measure. Hence, or otherwise, find the modulus and argument of z^2.

[9 marks]

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SECTION B

9. Find two values of θ between 0 and $\pi/2$ radians satisfying the equation

$$8\cos^2(\theta) = 11 - 10\sin(\theta).$$

[7 marks]

Using the identity $\sin(2A) = 2\sin(A)\cos(A)$ or otherwise, find *all* the solutions for the angle A in the range $0^0 \le A \le 360^0$ which satisfy the following equation

$$\sin(2A) + \sin^2(A) = 0.$$

[8 marks]

10. (i) On separate diagrams sketch the curves $y = 2e^{-x}$ for real x, and $y = \log_e(x) - 1$ for $x > 0$.

[4 marks]

(ii) Solve the following equations:

$$\log_{27}(x) = \frac{1}{3}, \qquad \log_y(1024) = 10.$$

[4 marks]

(iii) A capacitor is charged through a resistor R (ohms) by connecting it and the resistor in series to an electrical cell. The total charge Q that accumulates on the capacitor after a time t (measured in seconds) is given by the following equation

$$Q = Q_0\left(1 - \frac{3}{4}e^{-t/RC}\right),$$

where C is the capacitance of the capacitor (in Farads) and Q_0 is a constant. What was the initial charge on the capacitor before it was connected to the circuit? If $R = 100$ ohms, and after 5 seconds the charge on the capacitor has risen to $Q = 3Q_0/4$ coulombs, what is the value of C? How long will it take for the charge on the capacitor to reach a value of $0.99Q_0$ coulombs?

[7 marks]

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11. (i) If α and β are the roots of the equation $-2x^2-5x+1=0$, find the values of a) $\alpha\beta$, b) $\alpha+\beta$, c) $\alpha^2+\beta^2$ and d) $(\alpha-\beta)^2$, *without determining the values of* α *and* β individually.

[8 marks]

(ii) Plot a table of the values of the following cubic polynomial

$$p(x)=-4x^3+8x^2+11x-15,$$

for $x=-3,-2,-1,0,1,2,$ and 3. Sketch the curve of the polynomial, and find all the roots of $p(x)=0$.

[7 marks]

12. (i) A complex number *z* has modulus one and argument $\pi/3$. Express each of the following complex numbers in the form $a+b\mathrm{i}$ (where *a* and *b* are real):

$$z,\ z^2,\ z^3,\ \frac{1}{z},$$

and plot them on the Argand diagram.

[10 marks]

(ii) If $w=3+2\mathrm{i}$ is a root of the quadratic equation

$$2\mathrm{i}w^2+(\mathrm{i}-z)w+3-10\mathrm{i}=0,$$

calculate the value of the unknown complex number *z* in the form $a+\mathrm{i}b$.

[5 marks]