PAPER CODE NO.
MATH 013

THE UNIVERSITY *of* LIVERPOOL

JANUARY 2005 EXAMINATIONS

Bachelor of Engineering : Foundation Year
Bachelor of Science : Foundation Year

MATHEMATICAL METHODS

TIME ALLOWED : Three Hours

INSTRUCTIONS TO CANDIDATES

You may attempt all questions. All answers to Section A and the best THREE answers to Section B will be taken into account.
Numerical answers should be given correct to four places of decimals.

SECTION A

1. Determine the radian measure of the angle α of 480^0, expressed as a rational multiple of π.
The formula for $\cos(A-B)$ states that

$$\cos(A-B)=\cos(A)\cos(B)+\sin(A)\sin(B).$$

Using this formula or otherwise find the exact value for $\cos(\alpha)$, *without using tables or a calculator*. (Show all your working.)
Hence determine all the angles θ, in the range $[-360^0, 360^0]$ satisfying $\cos(\theta)=\cos(\alpha)$. Your answers can be expressed in either degrees or radians .

[6 marks]

2. Sketch the graph of $y=\sin(x)$ in the range $0 \le x \le 2\pi$. Determine numerically the solutions of both $\sin(x)=0.75$ and $\sin(x)=-0.75$ in the same range.

[9 marks]

3. Find the domain of *x* for which *both* the functions $\log_4(x)$ and $\log_4(2x-14)$ are defined.
Find the only valid solution to the equation

$$\log_4(2x-14)+\log_4(x)=2 .$$

[7 marks]

4. You are given the values of $\log_{10}(15)=1.176091$ and $\log_{10}(5)=0.698970$, correct to six decimal places. Obtain the values of the following

$$\log_{10}(75), \qquad \log_{10}(3), \qquad \log_{10}(125),$$

without using tables or a calculator, correct to four decimal places. (Show all your working. Hint: in last part $5^3=125$.)

[6 marks]

5. Write down the first six rows of Pascal's triangle. Hence or otherwise find the coefficient of x^3 in the expansion of

$$(2-x)^5 .$$

[6 marks]

6. Let $q(x)$ be the quadratic function $q(x) = x^2 + x - 6$. Determine the zeros of $q(x)$ and the position and nature of its turning point. Hence sketch the graph of $q(x)$.

[7 marks]

7. Express the rational function $f(x)$ in partial fractions, where

$$f(x) = \frac{2x - 21}{(x+2)(x-3)}.$$

[5 marks]

8. Express the complex number

$$z = \frac{-7 - 4\mathrm{i}}{-3 + 2\mathrm{i}}$$

in the form $z = a + b\mathrm{i}$.
Determine numerically the modulus and argument of z. The argument should, preferably, be expressed in radian measure. Hence, or otherwise, find the modulus and argument of z^2.

[9 marks]

SECTION B

9. Find two values of θ between 0 and π radians satisfying the equation

$$3\sin^2(\theta)+\cos(\theta)=1.$$

[7 marks]

Using the identity $\cos(A-B)=\cos(A)\cos(B)+\sin(A)\sin(B)$ express $\cos(x)+2\sin(x)$ in the form $R\cos(x-\alpha)$, where R is positive and α is an acute angle. Hence or otherwise solve the equation

$$\cos(x)+2\sin(x)=1.52,$$

for x in the interval [0^0 , 360^0].

[8 marks]

10. (i) On separate diagrams sketch the curves $y=e^x+1$ for real x, and $y=-\log_e(x)$ for $x>0$.

[4 marks]

(ii) Solve the following equations:

$$\log_3(81)=x, \qquad \log_y(32)=5.$$

[4 marks]

(iii) A new species of ladybird has been recorded in the U.K., and entomologists are keen to chart its population growth. Observations suggest that the number of new ladybirds N is growing according to the formula

$$N=1000\left(\beta+e^{kt}\right),$$

where t is the time in years, and β and k are constants. Initially there were believed to be only 2000 new ladybirds across the country. Show that $\beta=1$. After 2 years new observations suggest that the population has reached 6000. Determine the value of k. If this rate of growth continues, estimate in how many years will it take the population to reach one million $(N=1{,}000{,}000)$?

[7 marks]

11. (i) If α and β are the roots of the equation $3x^2 - x + 4 = 0$, write down the values of a) $\alpha\beta$, b) $\alpha + \beta$, c) $\alpha^2 + \beta^2$ and d) $(\alpha - \beta)^2$, *without determining the values of* α *and* β individually.

[8 marks]

(ii) Plot a table of the values of the following cubic polynomial

$$p(x) = 4x^3 - 20x^2 + 17x + 14,$$

for $x = -2, -1, 0, 1, 2, 3$ and 4. Sketch the curve of the polynomial, and find all the roots of $p(x) = 0$.

[7 marks]

12. (i) A complex number z has modulus one and argument $\pi/3$. Express each of the following complex numbers in the form $a + b\mathrm{i}$:

$$z,\ z^2,\ z^3,\ \frac{1}{z},$$

and plot them on the Argand diagram.

[10 marks]

(ii) Calculate the complex number z in the form $a + \mathrm{i}b$, if

$$z = \frac{(1+4\mathrm{i})(1-\mathrm{i})}{(2+\mathrm{i})3\mathrm{i}}.$$

[5 marks]