SECTION A

1. An athlete trains by running around a circular track of radius 70 metres at the constant speed of 6.5 metres per second. Through how many radians does he move in 10 seconds? After 6 minutes of running his coach tells him to stop when he next reaches his starting point. How much further does he have to run?

[7 marks]

2. Sketch the graph of $y = \cos(x)$ in the range $-\pi \leq x \leq 4\pi$.
Find all positive solutions of $\cos(x) = 0.84230$, where x does not exceed 10 radians.

[8 marks]

3. You are given the values: $\ln(12) = 2.484907$ and $\ln(3) = 1.098612$, correct to 6 places of decimals. Obtain values of the following

$$\ln(36),\ \ln(6),\ \ln(4),$$

without using tables or a calculator, correct to 4 places of decimals.
You should show your numerical working.

[6 marks]

4. Sketch separate graphs of the functions

$$y = e^{x} \ \text{ and } \ y = e^{-x}.$$

How are the shapes of these graphs related ?

[5 marks]

5. Find the coefficient of x^4 in the expansion of

$$2\left(3x^3 - 5x^{-2}\right)^3 + \left(2 + 3x^2\right)^4.$$

[7 marks]

6. Sketch the graph of the quadratic function $q(x) = 2x^2 - 5x - 7$.
Determine the zeros of $q(x)$ and the position of its minimum.

[7 marks]

7. Express the rational function $f(x)$ in partial fractions, where

$$f(x) = \frac{2x - 5}{(x-1)(x+2)}.$$

[7 marks]

8. Express the complex number

$$z = \frac{6 + 8i}{1 - i}$$

in the form $z = a + ib$.
Determine numerically the modulus and argument of z. The argument should be given in radian measure.
Deduce the argument of z^2.

[8 marks]

SECTION B

9. Assuming the *Difference Formula* for the sine function :

$$\sin(x-y)=\sin(x)\cos(y)-\cos(x)\sin(y),$$

show that $\sin(x-\frac{3\pi}{2})=\cos(x)$, for all x.
Express $4\sin(x)-3\cos(x)$ in the form $A\sin(x-\phi)$, where the phase angle ϕ is acute and $A>0$. The angle ϕ should be expressed numerically in radians.
Hence solve the equation

$$4\sin(x)-3\cos(x)=\frac{5}{\sqrt{2}},$$

where x is an obtuse angle. Comment on the case when the right hand side of this equation is replaced by $5\sqrt{2}$.

[15 marks]

10. (i) Use logarithms to solve the equation :

$$7^{3x-1}=3^{x+1},$$

for x. [6 marks]

(ii) Solve the equation

$$4^{2-3y}=8^{4y-5},$$

expressing y as an exact fraction. [4 marks]

(iii) Determine the number of digits possessed by 51^{75}, when expanded.

[5 marks]

11.(i) The quadratic $x^2 + 5x - 7$ has roots α, β. Write down the values of $\alpha + \beta$ and $\alpha\beta$.
Without calculating α, β, find the values of

$$\alpha^2 + \beta^2, \quad (\alpha - \beta)^2.$$

[7 marks]

(ii) Sketch the graph of the cubic polynomial

$$p(x) = 2x^3 - 9x^2 + 7x + 6.$$

Find all the roots of $p(x) = 0$. [8 marks]

12. A complex number z has modulus one and argument $\frac{\pi}{6}$. Express each of the following complex numbers in the form $a + ib$:

$$z,\ z^2,\ z^3,\ z^4,\ \frac{1}{z},\ z - \frac{1}{z},$$

and plot them on an Argand diagram.
[Hint: You may assume that $\sin(\frac{\pi}{6}) = \frac{1}{2}$.]

[15 marks]