SECTION A

1. Simplify:

(a) $\dfrac{a^9b^{-3}c^8}{a^7(bc^2)^4}$ (b) $\dfrac{16x^2-1}{16x^2+8x+1}$.

[4 marks]

2. Write $\dfrac{1}{y-7}-\dfrac{7}{y^2-7y}$ as a single fraction, and simplify it as far as possible.

[4 marks]

3. Solve the following quadratic equations:

(a) $x^2-x-72=0$ (b) $8x^2-2x-21=0$.

[4 marks]

4. Sketch the graph of each of the functions:

(a) $y=3x-6$ (b) $y=x^2+6x+8$ (c) $y=|x^2+6x+8|$.

[7 marks]

5. Given that $f(x)=\dfrac{2x+7}{1-3x}$, obtain an expression for the inverse function $f^{-1}(x)$.

[3 marks]

6. (a) Find the sum of the geometric series $\sum_{n=1}^{5}(-5)^n$.

(b) Write down the formula for the sum of the infinite geometric series $\sum_{n=1}^{\infty} ar^{n-1}$ with first term a and common ratio r, when $|r|<1$.

Hence show that $\sum_{n=1}^{\infty}\left(\frac{7}{10}\right)^n=\frac{7}{3}$.

[6 marks]

7. Evaluate the following limits:

(a) $\displaystyle\lim_{n\to\infty} \frac{2n^2 - 7n}{5n^2 + 6n - 1}$ (b) $\displaystyle\lim_{x\to 4} \frac{x^2 - x - 12}{x^2 - 16}$.

[4 marks]

8. Differentiate with respect to x

(a) $(3x - 8)^7$ (b) $(x^4 + 3)^{\frac{5}{4}}$ (c) $x^8 \cos x$.

[8 marks]

9. Write down the equation of the tangent line to the curve $y = 3x^4 + 2$ at the point where $x = -1$.

[3 marks]

10. Find the indefinite integrals:

(a) $\displaystyle\int (6 - 3x^5 - \sin x)\, dx$

(b) $\displaystyle\int e^{-5x}\, dx$.

[6 marks]

11. Evaluate the definite integrals:

(a) $\displaystyle\int_0^{\pi/14} \cos 7x\, dx$

(b) $\displaystyle\int_3^6 \frac{5}{5x - 14}\, dx$ [Substitute $u = 5x - 14$].

[6 marks]

SECTION B

12. The function $y = y(x)$ satisfies the equation

$$3x^3 + xy^2 - 2y^4 = -6.$$

(i) By differentiating both sides of this equation with respect to x find an equation relating x, y and $\dfrac{dy}{dx}$.

Hence show that $\dfrac{dy}{dx} = \dfrac{9x^2 + y^2}{8y^3 - 2xy}$.

(ii) Use this to show that the tangent line to the curve

$$3x^3 + xy^2 - 2y^4 = -6$$

at the point where $x = -1$ and $y = 1$ has equation $y = x + 2$.

(iii) Show that this line meets the curve with equation

$$y = x^2 + 5x + 6$$

in exactly one point.

[15 marks]

13. Differentiate the functions:

(a) $\ln(5 - \sin x)$ (b) $e^{5x+8}(7 - 3x)$

(c) $5x^7 - 10 - \cos^4 x$ (d) $\dfrac{2x^4 - 7}{\sin^3 x}$.

[15 marks]

14. (i) Find the stationary points and the inflection point of the function $f(x) = x^3 + x^2 - 12x$, in each case giving the values of x and $f(x)$ to 2 decimal places.

Determine also the nature of the stationary points.

(ii) Find the three points at which the curve $y = x^3 + x^2 - 12x$ crosses the x-axis.

(iii) Using the information from (a) and (b), sketch the curve $y = x^3 + x^2 - 12x$.

(iv) Calculate the total area bounded by the curve and the x-axis.

[15 marks]

15. Find the indefinite integrals:

(a) $\displaystyle\int x^5 \cos(x^6 + 4)\, dx$ [Substitute $u = x^6 + 4$]

(b) $\displaystyle\int \tan^8 x \sec^2 x \; dx$ [Substitute $t = \tan x$].

Evaluate the definite integral:

(c) $\displaystyle\int_0^{\sqrt{2}/5} \frac{dx}{\sqrt{4 - 25x^2}}$ [Substitute $x = \dfrac{2}{5}\sin t$].

[15 marks]