All questions carry equal weight. If a question has several parts, each part will carry equal weight. The best **eight** answers will be taken into account.

1. (a) Simplify the following:

(*i*) $5p - 3rs + 5p + 3rs$; (*ii*) $p^2 + 2pr - 2p^2 - 3pr$;

(*iii*) $2p + 6rs + 2rs - 4p$; (*iv*) $2(x + y) - (x - y)$.

(b) Simplify

(*i*) $\frac{2^{27}}{2^{23}}$; (*ii*) $\frac{4^7}{8^4}$.

2. Simplify

(*i*) $\frac{x}{x+1} - \frac{x}{x-1}$; (*ii*) $\frac{7}{x^2} + \frac{5}{x}$;

(*iii*) $\frac{x}{(x-1)(x+1)} - \frac{2}{(x-1)(x+2)}$ (*iv*) $\frac{x}{(x-1)} + \frac{1}{(1-x)}$.

3. (a) Work out $\frac{x}{y} + \frac{-y}{x}$; $\frac{x}{y} - \frac{-y}{x}$; $\frac{x}{y} \times \frac{-y}{x}$ and $\frac{x}{y} \div \frac{-y}{x}$.

(b) Solve the inequalities:

(*i*) $-2x + 3 > -1$ and (*ii*) $x - 1 > 2x + 1$.

4. (a) Factorise the following quadratics and hence find their solutions:

(*i*) $x^2 - x - 12$; (*ii*) $2x^2 + 5x + 2$.

(b) Use the formula to solve the following quadratics:

(*i*) $x^2 + 4x + 1 = 0$; (*ii*) $3x^2 + 2x - 5 = 0$.

5. (a) Let $y = 2\ \log_a 12 - log_a 9$. Express y as a single logarithm and find y when $a = 2$.

(b) A coin is tossed 4 times. Using "T" for tails and "H" for heads, write out all possible outcomes. How many of these outcomes have exactly two heads and two tails?

6. (a) In a class of 32 students, 16 study French and 15 study German. If 3 students study both French and German, how many students study neither language?

(b) Decide which of the following sets are equal:
$A = \{n \text{ in } \mathbf{Z} : 0 < n < 8 \text{ and } n^2 < 2n + 1\}$; $B = \{n \text{ in } \mathbf{Z} : n^3 = n\}$;
$C = \{0, 1, 2\}$; $\quad D = \{n \text{ in } \mathbf{Z} : n^2 < 2\}$.

7. (a) Use the binomial theorem to find:
$(i)\ \ (2x + y)^4$ and $(ii)\ \ (x - 2y)^3$.

(b) Prove, by mathematical induction that for all natural numbers n

$$1 + 3 + 5 + \cdots (2n - 1) = n^2.$$

8. (a) A race has 8 competitors. How many possible results (first, second, third) are there?

(b) In how many ways can a committee of 4 be formed from 10 people?

(c) How many distinct arrangements of the four letters H,A, L and L are there?

(d) In how many ways can 8 identical coins be distributed between 6 children?

9. Write down the truth tables for the expressions

$$(i)\quad (p \to q) \to (q \to p); \qquad (ii)\quad \neg(p \to q) \to (\neg p \vee \neg q)$$

and decide whether either is a tautology.

10. In the set of all integers, let $p(x)$ be the predicate "$x > 1$" and $q(x)$ be the predicate "$x < 6$". Decide which of the following are true and which are false

(i) $\forall x(p(x) \vee q(x))$; (ii) $\forall x(p(x)) \vee \forall x(q(x))$;

(iii) $\exists x(p(x) \wedge q(x))$; (iv) $\exists x(p(x)) \wedge \exists x(\neg q(x))$.

11. Draw a graph to satisfy each of the following specifications or indicate why it is impossible to do so

A simple graph with 4 vertices and 3 edges;

A graph with 4 vertices and 7 edges;

A simple graph with 4 vertices and 7 edges;

A connected graph with 4 vertices and 2 edges.

12. Let Γ be a graph with v vertices and e edges. Write down a formula relating e to the degrees of the graph at each vertex of Γ. Now suppose that Γ is a tree, write down a relationship between e and v. Suppose that Γ is a tree with 4 vertices of degree 1, 1 vertex of degree 2 and k vertices of degree 4. Determine k and draw Γ

13. Given the 2×2 matrix

$$A = \begin{pmatrix} -1 & -1 \\ 0 & 1 \end{pmatrix}$$

calculate A^2, $\det A$ and A^{-1}. What is A^3?