1. Define the terms: *mixed strategy; behavioural strategy.*

In a game for two players, I and II, five cards are placed in a hat, two with 2 on them and three with 0 on them. The players stake is 50p. each. I draws a card from the hat without showing II. He can either ’raise’, ’quit’ or ’show’. If he ’raises’, he puts 100p. in the kitty; if he ’shows’, he shows the card to II, puts the card back into the hat and adds 50p. to the kitty. For either of these he gets 2 points plus the points on the card. If he quits he loses his kitty and the game ends.

It is now II’s turn. II can either ’raise’, ’buy’ or ’quit’. The procedures for ’raising’and ’quitting’ are the same as for I, except that he draws a card after he ’raises’. If he ’buys’, he puts 50p. into the kitty and then draws a card and looks at it. He can then ’raise’ or ’quit’, using I’s procedures.

The player with the most points wins the kitty. If the points are equal the kitty is shared.

Draw the extensive form for this game (you need only provide one example of each payoff which requires a different type of calculation.). Outline examples of a behavioural strategy, which include every possible action, for both I and II.

2. Explain the following terms: *perfect information, incomplete information, perfect recall, priors.*

A two-player game, for I and II, has the following pay-off matrix:

$$
\begin{array}{rcc}
 & \mid II(i) > & \mid II(ii) > \\
\mid I(i) > & \{(1,2) & (4,4)\} \\
\mid I(ii) > & \{(2,6) & (2,5)\}
\end{array}
$$

What are the Nash equilibria of this game? Give examples of the extensive form of this game:-(a) where both players play simultaneously: (b) where player I plays first and his action is known to II: (c) where player II plays first and his action is known to I. Find the *solution(s)* in each case, giving your reasoning.

3. Explain what is meant by the terms *subgame perfect equilibrium,* and *backwards induction.*

This is a game for two players, I and II, who take it in turns to take discs from the table. Players cannot pass. Six discs are placed on the table. The player who moves last loses. For his first move I can remove one or three discs. For all other moves one or two discs may be removed.

Draw the game tree for this game and find all the subgame perfect equilibria.

4. Explain the terms *belief* , *assessment* and *sequential equilibrium.*

An art dealer, who is an art expert, has come into the possession of a set of 'old master' paintings. Other experts, who have had a cursory look at the paintings estimate that half the paintings are genuine (worth \$ 10,000) and half are fakes (worth \$ 500.) If he decides to sell one of the paintings he sets the price at \$ 5,000. The buyer can either buy, refuse to buy (so that the dealer loses \$ 100) or have it authenticated by a museum. The latter costs \$ 2,000 and if the buyer then does not buy the dealer loses \$ 4,000 if it is a fake, as his reputation has been tarnished.

Draw the game-tree for the game of selling one painting and describe it as a signalling game. Suggest a possible sequential equilibrium for this game and briefly justify your suggestion qualitatively.

5. Define the terms *superadditivity* , *convex* and *monotonic* when applied to the characteristic function of a cooperative n-person game. Define also the term *dummy player.*

Consider a market with three buyers, A,B and C, and three sellers I,II and III. Each of the latter has one identical vase for sale, which they are willing to sell at \$8., \$10. and \$14. respectively. A,B and C are respectively willing to spend \$13, \$11 and \$7 to buy such a vase. Consider this two-sided market as a 6-person cooperative game and find its characteristic function. Hence find the core and the limits on the price of vases in this market.

6. Compare the advantages and disadvantages of the core and the Shapley value as solutions for a cooperative n-person game.

Four people, A, B, C and D form a committee and have 48, 24, 18 and 10 votes respectively. To ensure the passing of a resolution, m votes are required. Find the minimum winning coalitions when (i) $m = 56$, and (ii) $m = 51$. Treating these situations as simple games, find the core and the Shapley value in each case.

Use these results as illustrations of the advantages and disadvantages you discussed above.

7. (a) Write down the conditions for an evolutionary stable strategy (E.S.S.) for a symmetrical game. In what way does this differ from the conditions for a Nash equilibrium in such a game?

Write down also the payoffs in the game 'War of Attrition' and discuss the biological reasoning behind this model. State the E.S.S. for this game?

(b) Write down the conditions for the E.S.S.s of an asymmetrical game. Show that the game with payofffs

$$\begin{array}{ccc} & \mid II(i) > & \mid II(ii) > \\ \mid I(i) > & \{(2,2) & (4,4)\} \\ \mid I(ii) > & \{(0,0) & (10,-2)\} \end{array}$$

does not have an E.S.S.