PAPER CODE NO. COMP309

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THE UNIVERSITY
of LIVERPOOL

JANUARY 2004 EXAMINATIONS

Bachelor of Arts : Year 3
Bachelor of Science : Year 3

EFFICIENT SEQUENTIAL ALGORITHMS

TIME ALLOWED : Two Hours and a Half

INSTRUCTIONS TO CANDIDATES

Answer four questions only. Each question is marked out of 25.

If you attempt to answer more than the required number of questions (in any section), the marks awarded for the excess questions will be discarded (starting with your lowest mark).

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1. (a) Describe the 4 steps in the cookbook recipe for dynamic programming. **(6 marks)**

   (b) Give a pseudo-code description for a recursive procedure to compute $m[i, j]$, the function giving the minimum number of multiplications to multiply matrices A_i through to A_j. **(8 marks)**

   (c) Run the MATRIX-CHAIN-ORDER algorithm on the sequence 2, 3, 4, 5, 6, 10. What is the optimal number of multiplications needed? **(11 marks)**

2. (a) Give a description of Lempel-Ziv compression algorithm. **(10 marks)**

   (b) The string aaabaaaabbaaaa has to be compressed using Lempel-Ziv compression. Write down the encoded string, assuming that the initial string table is

a	#1
b	#2

   and that a block code of length three is used to encode the symbols in the table (#1 corresponds to 000, #2 to 001, #3 to 010, and so on). **(6 marks)**

   (c) Suppose #1 = 0 and #2 = 1. What is the length of the Huffman code for the given string? Assuming a variable length prefix code is used for the symbols in the string table (e.g. #1 corresponds to 0, #2 to 10, #3 to 110, and so on), what is the length of the Lempel-Ziv compressed string? **(9 marks)**

3. (a) Define the terms *polygon*, *convex polygon*, and *convex hull*. **(5 marks)**

   (b) Describe (either using an English description or pseudo-code) GRAHAM-SCAN algorithm for computing the convex hull of a set of points. **(7 marks)**

   (c) Run GRAHAM-SCAN algorithm on the following list of points:

$$(4, 4), (5, 2), (2, 1), (2, 3), (3, 3), (6, 6), (1.5, 5), (1, 5).$$

   List all intermediate stack configurations. **(13 marks)**

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4. On Savitch's theorem.

   (a) Give an argument proving that 2^n is space constructible. **(6 marks)**

   (b) Give a definition of NSPACE(2^n). **(5 marks)**

   (c) State the equality entailed by Savitch's theorem for the complexity class NSPACE(2^n). **(6 marks)**

   (d) Give an argument supporting the validity of Savitch's theorem. Your answer should describe the way in which a non-deterministic space bounded Turing machine can be simulated by a deterministic one. **(8 marks)**

5. (a) Give the definition of *absolute performance guarantees*. **(3 marks)**

   (b) Give the definition of $r(n)$-approximation algorithm. **(4 marks)**

   (c) Define the BINPACKING problem. Your definition should express the fact that BINPACKING is an optimisation problem (i.e. all four components in the definition of optimisation problem must be defined in the specific case). **(5 marks)**

   (d) Consider the so called *First Fit* greedy heuristic which goes down the list of items and fits each item into the *first* bin where it will fit. If no open bin has any room for the current item a new bin is opened and the current item is placed in it.

      i. Run the algorithm on an instance consisting of
         - 6 items of size $\frac{1}{7} + \frac{1}{1000}$
         - 6 items of size $\frac{1}{3} + \frac{1}{1000}$
         - 6 items of size $\frac{1}{2} + \frac{1}{1000}$

         (presented in this order). Describe the solution returned by an algorithm following the First Fit heuristic. **(7 marks)**

      ii. How many bins are used by an optimal solution? **(6 marks)**