PAPER CODE NO. COMP304	EXAMINER : Wiebe van der Hoek DEPARTMENT : Computer Science Tel. No. 0151 794 7480

THE UNIVERSITY
of LIVERPOOL

JANUARY 2004 EXAMINATIONS

Bachelor of Arts : Year 3
Bachelor of Engineering : Year 3
Bachelor of Science : Year 3

Knowledge Representation and Reasoning

TIME ALLOWED : Two Hours and a half

INSTRUCTIONS TO CANDIDATES

Answer **four** questions only.

If you attempt to answer more questions than the required number of questions (in any section), the marks awarded for the excess questions will be discarded (starting with your lowest mark).

1. (Knowledge Representation and Formalisms)

   (a) Explain why formal tools, and in particular logics, are useful for knowledge representation and reasoning. Refer to the three parts that make up a logic. (6 marks)

   (b) Give an example of a sentence in natural language that is ambiguous, and use a logical language to disambiguate. (6 marks)

   (c) Explain what is meant by the logical omniscience problem, when using modal logic for reasoning about knowledge. Give an example of (an effect of) this problem. Relate it to the K-axiom, and give three other modal principles that are valid, and which are logical omniscience properties. Give an argument why logical omniscience is not really a problem. (13 marks)

2. (Modal Logic)
   Let the Kripke model $M = \langle W, R, I\rangle$ be given by

$$
\begin{aligned}
W &= \{1,2,3,4\}\\
R &= \{(1,2),(1,3),(2,2),(3,4)\}\\
I &= \{(p,\{2,3,4\})\}
\end{aligned}
$$

   (a) Draw the labelled graph corresponding to M, that is, draw the Kripke model M. (5 marks)

   (b) Give formal derivations which determine whether the following are true:
   (*i*) $M, 1 \models \Box p \rightarrow p$
   (*ii*) $M, 1 \models \Box\Diamond p$
   (12 marks)

   (c) Argue whether $M, 1 \models \Diamond(p \rightarrow \Box\bot)$ (8 marks)

3. (Description Logic)
   Let the knowledge base Γ be given by the following set of assertional and terminological sentences.

$\text{Young} \doteq \neg\text{Old}$	$(\text{gregor}, \text{sprinter}) : \text{hasCar}$
$\text{Vital} \doteq \text{Old} \sqcap \exists\text{hasCar.Sport}$	$\text{sprinter} : \text{Sport}$
$\text{Yup} \doteq \text{Young} \sqcap \exists\text{hasCar.Sport}$	$\text{gregor} : \text{Old}$
$\text{Target} \doteq \text{Vital} \sqcup \text{Yup}$	$\text{blazer} : \text{Sport}$

   (a) Give the expanded TBox of the knowledge base Γ. (5 marks)

   (b) Give a formal derivation of the negation normal form of $\neg\text{Vital}$ with respect to the TBox of Γ. (8 marks)

   (c) Give a formal derivation which determines whether gregor is an element of the concept Vital, that is, determine by a formal derivation whether gregor is an instance of Vital. (12 marks)

4. (Epistemic Logic: Semantics)
We model the muddy children puzzle for the case of three children, all being muddy. Use as states in the model $(x, y, z) \in \{0, 1\}^3$, where for instance $(1, 1, 0)$ denotes the (hypothetical) situation in which children 1 and 2 are muddy, but child 3 is not. Hence, the real world is denoted by $w = (1, 1, 1)$.

   (a) Describe and make a drawing of the Kripke model that represents this situation. Clearly indicate the accessibility relations. Explain why your drawing represents the situation. (16 marks)

   (b) Now the father announces that at least one child is muddy. Draw the Kripke model that represents this new situation. (9 marks)

5. (Epistemic Logic)

   (a) Give a formal derivation within **S5** of the following formula:

$$K\varphi \leftrightarrow \neg K \neg K\varphi.$$

   Use the format: "name, property, reason" in your derivation. (12 marks)

   (b) We denote Implicit (or Distributed) Knowledge with the operator I, and Common Knowledge with C. Show that if $m \geq 2$ we do not have

$$S5_m \models I\varphi \rightarrow C\varphi$$

   (13 marks)