PAPER CODE NO. COMP304	EXAMINER : Prof. W. van der Hoek and Dr. U. Hustadt DEPARTMENT : Computer Science Tel. No. 7480 and 7431

THE UNIVERSITY
of LIVERPOOL

JANUARY 2003 EXAMINATIONS

Bachelor of Arts: Year 3
Bachelor of Engineering: Year 3
Bachelor of Science: Year 2
Bachelor of Science: Year 3
Bachelor of Science: Year 4

Knowledge Representation and Reasoning

TIME ALLOWED : Two Hours and a half

INSTRUCTIONS TO CANDIDATES

Answer **four** questions only.

If you attempt to answer more questions than the required number of questions (in any section), the marks awarded for the excess questions will be discarded (starting with your lowest mark).

1. State the so-called *knowledge principle* and give a critical account of it; discuss at least three critical points. (25 marks)

2. Let the Kripke model $\mathcal{M} = ((W, R), I)$ be given by

$$W = \{1, 2, 3, 4, 5\}$$
$$R = \{(1, 3), (2, 3), (3, 4), (3, 5)\}$$
$$I = \{(p, \{1, 4\}), (q, \{2, 5\})\}$$

   (a) Draw the labelled directed graph corresponding to $\mathcal{M}$. (5 marks)

   (b) Give formal derivations which determine at which worlds in the Kripke model $\mathcal{M}$, defined above, the formula $q \vee p$ is true. (6 marks)

   (c) Give a formal derivation which determines whether the formula $\Box\Diamond q$ is true at world 1 in the Kripke model $\mathcal{M}$ defined above. (14 marks)

3. Consider common knowledge, everybody's knowledge, and distributed knowledge in $\mathbf{S5I_{(m)}}$ with $m > 1$. Let p and q be propositional variables.

   (a) Give the relationship between the following formuals (if one is stronger than the other, indicate why, if not, give a countermodel):
   K_1p, Cp, p, Ep, Ip. (15 marks)

   (b) How would the results change if we took instead of p the formula K_1q? That is, give the relationship between the following formulas (if one is stronger than the other, indicate why, if not, give a countermodel):
   K_1K_1q, CK_1q, K_1q, EK_1q, IK_1q. (10 marks)

4. Consider a distributed system for three processors; the worlds in the Kripke model are thus all vectors $\mathrm{s} = (s_1, s_2, s_3)$, where s_i denotes the local state of processor i, and where for every $i \in \{1, 2, 3\}$ the accessibility relation R_i is defined by $R_i(\mathrm{s}, \mathrm{t}) \Leftrightarrow s_i = t_i$.
Suppose for the example that each processor is characterised by only one variable: t, v, k, respectively. Hence, we can denote the worlds as triples of variables (t, v, k). The variable t denotes time, v 'flag' and k colour. The variable t is a natural number ≥ 1, $v \in \{0, 1\}$ and $k \in \{\mathrm{g}, \mathrm{o}, \mathrm{r}\}$. Examples of primitive propositions in this context include $t = 7$, $v = 1$, $k = \mathrm{r}$, $t > 13$, etc.

   (a) Draw a picture of this distributed system, clearly denoting the accessibility relations. (10 marks)

   (b) Give formulas φ, α, and β such that β is not a tautology and

$$M, (1, 0, g) \models K_1\varphi \wedge \neg E\varphi \wedge E\alpha \wedge \neg C\alpha \wedge C\beta$$

(15 marks)

5. Let $\mathcal{K}$ be the knowledge base below

$$\begin{aligned} \text{male} &\doteq \neg\text{female} \\ \text{parent} &\doteq \text{person} \sqcap \exists\text{hasChild}.\top \\ \text{mother} &\doteq \text{parent} \sqcap \text{female} \\ \text{father} &\doteq \text{parent} \sqcap \text{male} \end{aligned} \qquad \begin{aligned} \text{tim} &: \text{person} \\ \text{tim} &: \neg\text{female} \\ \text{sue} &: \text{person} \\ (\text{tim}, \text{sue}) &: \text{hasChild} \end{aligned}$$

(a) Give the expanded TBox of the knowledge base $\mathcal{K}$. (4 marks)

(b) Compute the negation normal form of the concept $\neg$father with respect to the TBox of the knowledge base $\mathcal{K}$. (8 marks)

(c) Give a tableau derivation which determines whether tim is an element of the concept father. (13 marks)