PAPER CODE NO. COMP301	EXAMINER : **Mrs P Bench-Capon** DEPARTMENT : **Computer Science** Tel. No. **43697**

THE UNIVERSITY of LIVERPOOL

JANUARY 2001 EXAMINATIONS

Bachelor of Arts : Year 3
Bachelor of Science : Year 3

NETWORKS IN THEORY AND PRACTICE

TIME ALLOWED : Two Hours and a Half

INSTRUCTIONS TO CANDIDATES

Full marks will be awarded for complete answers to **FOUR** questions
Only the best **FOUR** answers will be taken into account

A figure sheet is provided for use in answering question 4. Please ensure that the Figure Sheet is attached securely to your answer book.

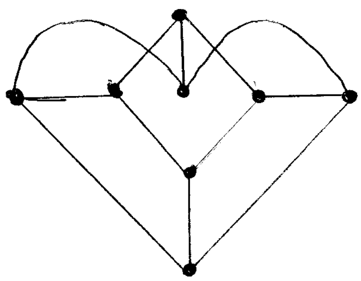
If you attempt to answer more than the required number of questions (in any section), the marks awarded for the excess questions will be discarded (starting with your lowest mark).

1(a) (i) Explain what it means to say a graph is *planar*. (2 marks)

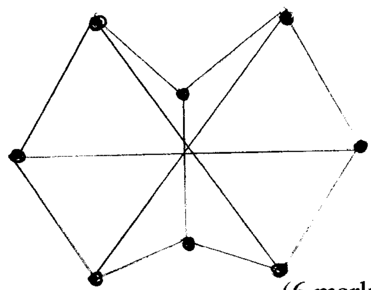
(ii) Any plane drawing of a connected planar graph obeys Euler's formula:
$v - e + f = 2$, where v, e and f are the number of vertices, edges and faces respectively. A graph is non-planar if and only if it contains a subgraph homeomorphic to K_5 or $K_{3,3}$.

State which, if any, of the following graphs G_1 , G_2 are planar.
For each that you believe to be planar, derive a plane drawing and verify Euler's formula. For any others, find a subgraph homeomorphic to K_5 or $K_{3,3}$.

G_1



G_2



(6 marks)

(b) (i) In an undirected graph, explain why the number of vertices of odd degree must be an even number. (3 marks)

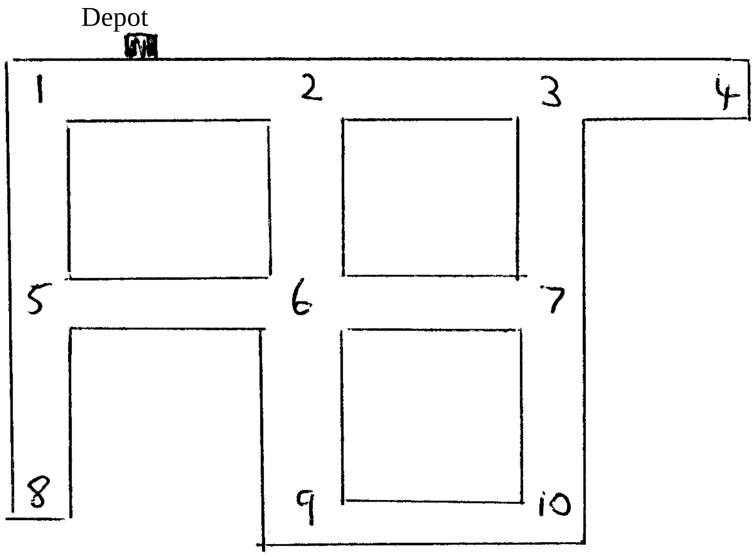
(ii) Leaflets are to be delivered throughout an undirected network of eight streets, with five junctions. The matrix $D = (d_{ij})$ gives the length d_{ij} of each street (edge) i j, and the matrix $L = (l_{ij})$ gives the length l_{ij} of the shortest chain between each pair of junctions (vertices) i j.

$$D = \begin{bmatrix} \infty & 14 & 12 & 4 & \infty \\ & \infty & \infty & 6 & 10 \\ & & \infty & 7 & 13 \\ \text{symmetric} & & & \infty & 5 \\ & & & & \infty \end{bmatrix} \qquad L = \begin{bmatrix} - & 10 & 11 & 4 & 9 \\ & - & 13 & 6 & 10 \\ & & - & 7 & 12 \\ \text{symmetric} & & & - & 5 \\ & & & & - \end{bmatrix}$$

Determine which edges must occur more than once in a minimal length Chinese Postman's Tour of the network.
Calculate the length of the tour, and draw the network including the necessary multiple edges.
(7 marks)

Q1 continues:-

1(c) The following network of twelve streets is to have its gutters cleared by a large vehicle, which is difficult to turn. The vehicle must go along each side of each street once, driving on the left, starting and finishing at the depot.
Junctions are labelled 1,2........10.

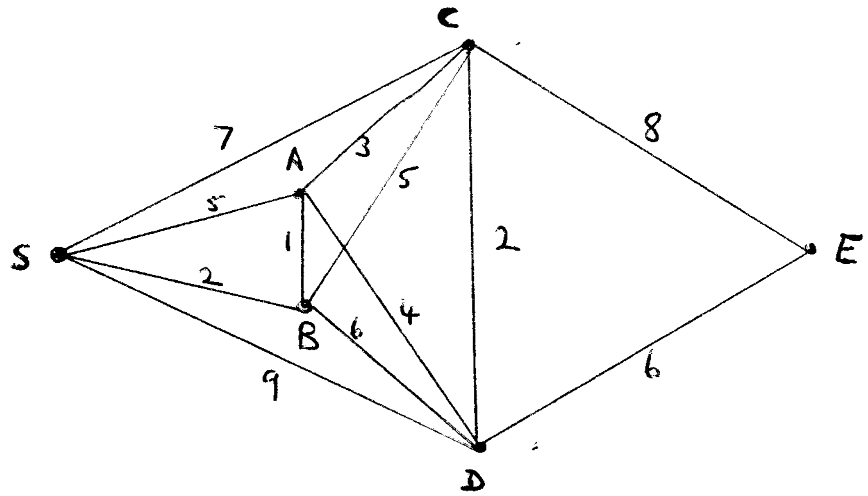


Turn penalties are assessed as : U turn: 6
right turn: 3
left turn: 1
straight on: 0

Determine the Turn Penalty Matrix for one junction of each of the four types:
T junction, crossroads, simple turn, dead end.
Hence find optimal routes through each junction type for the construction of an Euler circuit of the network with minimum total penalty.
Calculate this total penalty, and write down the circuit as a list of junctions visited, assuming the vehicle initially turns left out of its depot.

(7 marks)

2(a). The undirected network N below represents a network of footpaths, in a bad state of repair, connecting five ancient sites (vertices A to E) with a car park (vertex S). The Council wishes to restore some of the footpaths, to provide access to the sites, but cannot afford to restore them all.
The number alongside each edge represents the cost of repair.



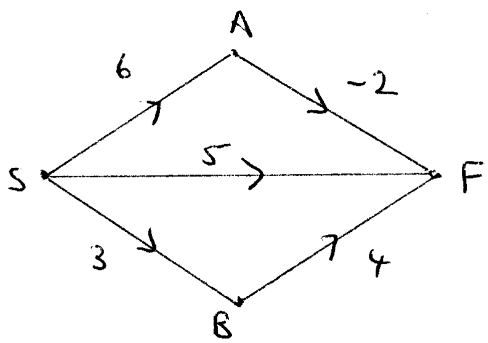
(i) Use Dijkstra's algorithm to find the cheapest distance from S to each of the other vertices of N.
Draw the cheapest route tree rooted at S, and calculate its total cost.

(8 marks)

(ii) The Council also wishes to consider the second cheapest route from S to E.
Describe how you would use the answer to (i) to find this.
You need not carry out the process. (4 marks)

(iii) As a cheaper option, the Council considers a Minimum Spanning Tree.
Give a brief description of Prim's Algorithm, and use the algorithm to construct a MST of N, starting with vertex S.
Calculate the total cost of repairing the footpaths in the MST. (8 marks)

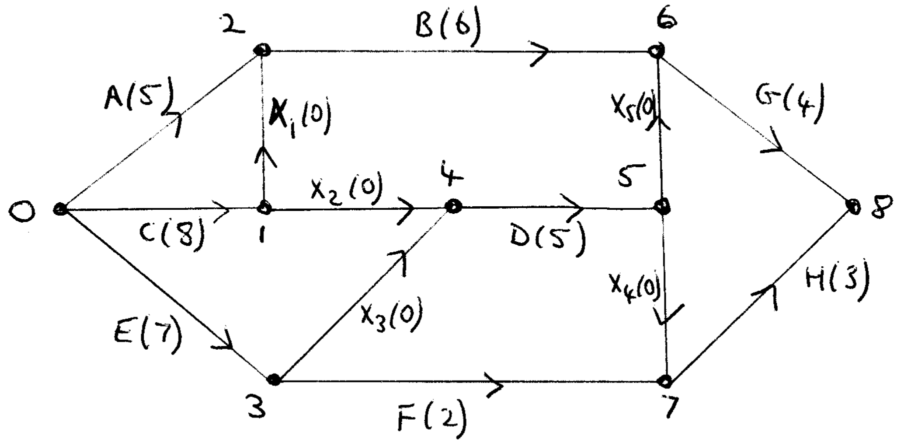
(b) In the following directed network N*, Dijkstra's algorithm would calculate the wrong shortest distance from S to F.
By inspection, state the correct shortest distance, and the answer that Dijkstra's algorithm would give.
Use Ford's algorithm to confirm the correct shortest distance, showing clearly how $d^{(2)}(F)$ is calculated.



(5 marks)

3. A project has been broken down into eight activities labelled A,B,…….H.
In the corresponding project network, given below, the number in brackets by each label represents the number of days that the activity is expected to take.

The vertices represent "events", the start and finish of activities.



(a) Perform *Forward Pass* to calculate the earliest time that the project can be completed.
(7 marks)

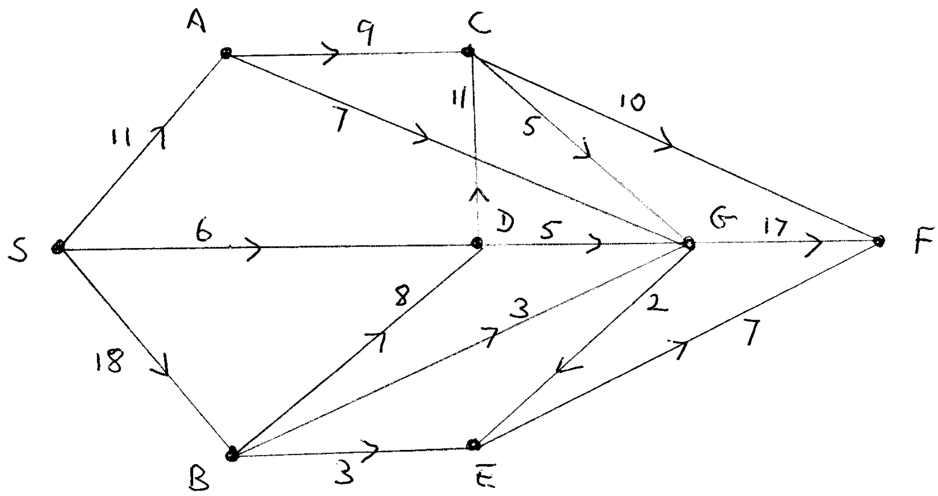
(b) Calculate the *Latest times* at which each event can occur, without delaying the finishing time, and determine the *Critical Path(s).*
(8 marks)

(c) Because of a shortage of workers, the completion of one of the activities B, D, or F must be delayed by 4 days. Find the *total float* of each of these activities, showing how it is obtained.
How should the manager deploy the workers to minimise the delay in completion of the whole project?
(4 marks)

(d) The activities X_1,$X_{2.}$ ......,X_5 are *dummy activities*. Explain what this means, and why dummy activities are necessary, with specific reference to X_2 and X_3.
(4 marks)

(e) Suppose a project network has three sinks: F_1, F_2 and F_3. Describe how you would modify the network to make it standard.
(2 marks)

4. In the network N given below, the number appearing alongside each arc represents its capacity.



(a) Define a *cut*, and find the *capacities* of these two cuts of N:
({S, A, B, E},{C, D, G, F}) and
({S, A, B, D, E, G}, {C, F}).
What can you deduce about the value of the maximum flow in N ?

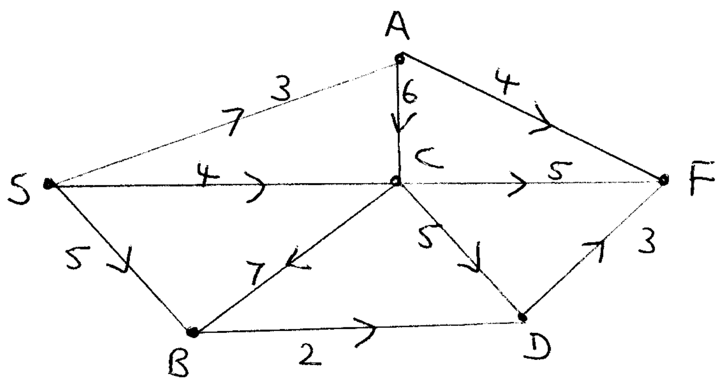
(4 marks)

(b) Using the Figure Sheet provided, use Berge's Superior Path method to derive a feasible flow in N. Use Diagram A on the Figure Sheet for your working, and Diagram B for your final answer. (7 marks)

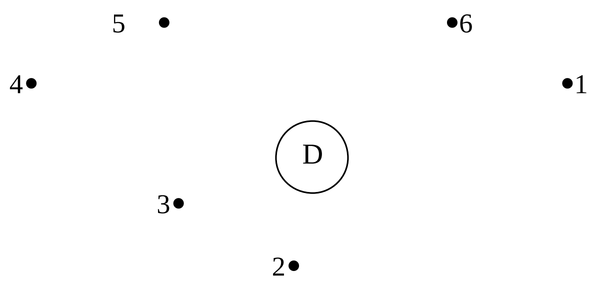
(c) Using Diagram C on the Figure Sheet, start with the flow you have found, and use the Ford-Fulkerson algorithm to determine a *maximal flow* in N from S to F, and a *minimal cut.* (10 marks)

(d) In a different network N* , given below, an upper limit of 8 is imposed on the flow through vertex C. Transform N* to an equivalent standard network.
The number alongside each arc represents its capacity.

(4 marks)



5. Six retail stores, denoted 1,2……6, are located approximately as shown around a depot, D, which supplies them with goods.



The distance in Km between each pair of stores, and from depot to store, is given in the symmetric matrix below.

	D	1	2	3	4	5	6
D	-	17	10	8	15	12	13
1		-	25	23	28	18	9
2			-	7	23	22	21
3				-	14	16	18
4					-	11	19
5						-	10
6							-

Three vehicles, each with capacity 20 tonnes, are available to deliver to the stores. The load for any store cannot be split between vehicles.
The weight of the order for store i is q_i, given by $\mathbf{q}$ = (4, 5, 7, 5, 13, 5).

(a) (i) Apply Phase 1 of the SWEEP method, with clockwise sweeping only, to the above data, to find a solution to the routing problem which minimises the total distance travelled by the vehicles.

(9 marks)

(ii) Draw the network for this minimal distance, labelling each arc with its direction, distance and the weight of the load to be carried along it.

(2 marks)

(b) (i) Apply the Clarke-Wright Savings Method to the same problem. (9 marks)

(ii) Draw the network found for this minimal distance, choosing directions for the arcs to minimise the weight of the load carried, where possible. Label it as in (a) (ii).

(3 marks)

(c) If only two vehicles were available, how could the stores be allocated between them? Suggest a reason why the three vehicle policy might be preferable.

(2 marks).

6(a) Five tasks are to be assigned to five people, so that each person performs one task. Estimates of the time t_{ij} hours, taken by person P_i to perform task T_j, are given in the matrix below.

	T_1	T_2	T_3	T_4	T_5
P_1	14	15	14	12	17
P_2	16	10	14	12	17
P_3	17	11	13	15	16
P_4	17	16	11	13	15
P_5	16	10	13	14	16

Use the Hungarian Method (Method of Lines) to determine the assignment of tasks to people which minimises the total number of person-hours taken.
Calculate this number of hours.
You may determine the lines by inspection.

(11 marks)

(b) Now suppose that the matrix represents a Travelling Salesman Problem.
Is the solution found in (a) feasible as a TSP solution? Give a reason for your answer.
What can you deduce from the solution to (a) about the value of an optimal solution to the TSP? Give a reason for your answer.

(4 marks)

(c) In the *Bottleneck* Assignment Problem, the greatest individual time to perform any task must be minimised. Show that an optimal assignment from the matrix (t_{ij}) is also optimal when t_{ij} is replaced by $t_{ij} - c$, where c is a constant.

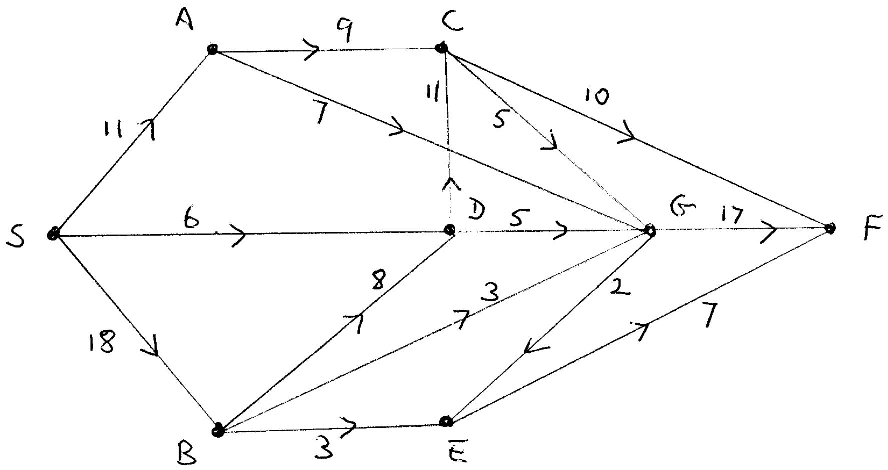
(4 marks)

(d) Leaving persons P_1 and P_2 assigned to the tasks found in (a), solve the Bottleneck Assignment Problem for the three remaining tasks and people.
State the minimum greatest individual time that you find.

(6 marks)

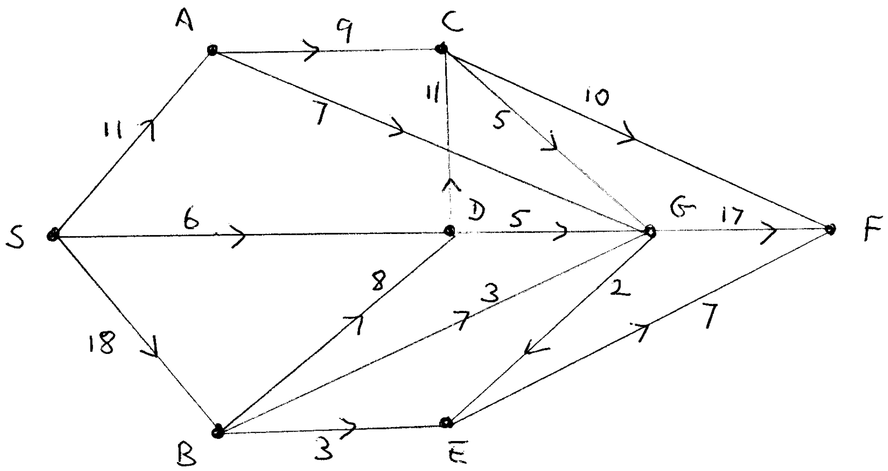
Figure Sheet for Question 4.

Diagram A: working for part (b)



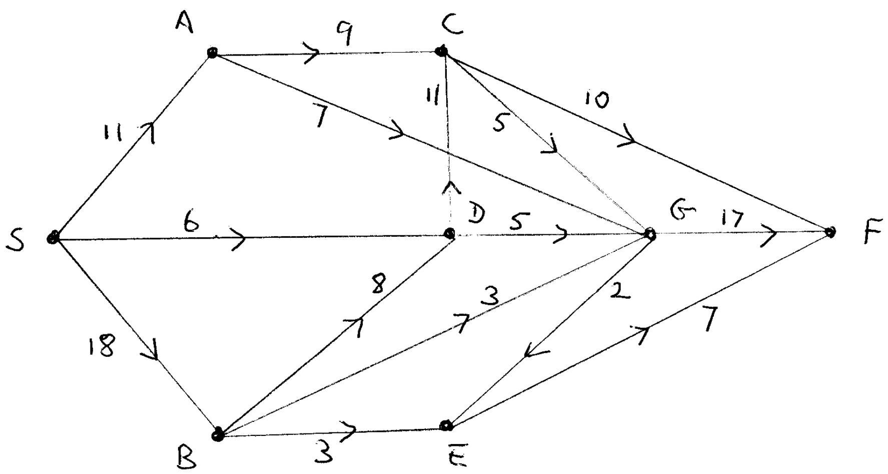
Capacities c_{ij} are shown

Diagram B: answer to part (b)



Capacities c_{ij} are shown. Add flows, to show f_{ij}/c_{ij}.

Diagram C: answer to part (c)



Capacities c_{ij} are shown. Add flows, to show f_{ij}/c_{ij}.