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THE UNIVERSITY
of LIVERPOOL

JANUARY 2004 EXAMINATIONS

Bachelor of Science : Year 2

Decision, Computation and Language

TIME ALLOWED : 2 Hours

INSTRUCTIONS TO CANDIDATES

Answer **all** questions in Section 1 and **two** questions from Section 2.

If you attempt to answer more questions than the required number of questions (in any section), the marks awarded for the excess questions will be discarded (starting with your lowest mark).

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Section 1

Answer **all** questions in this section.

1. Define what is meant by the following terms,

a. A **language** over an alphabet, Σ. **(3 marks)**

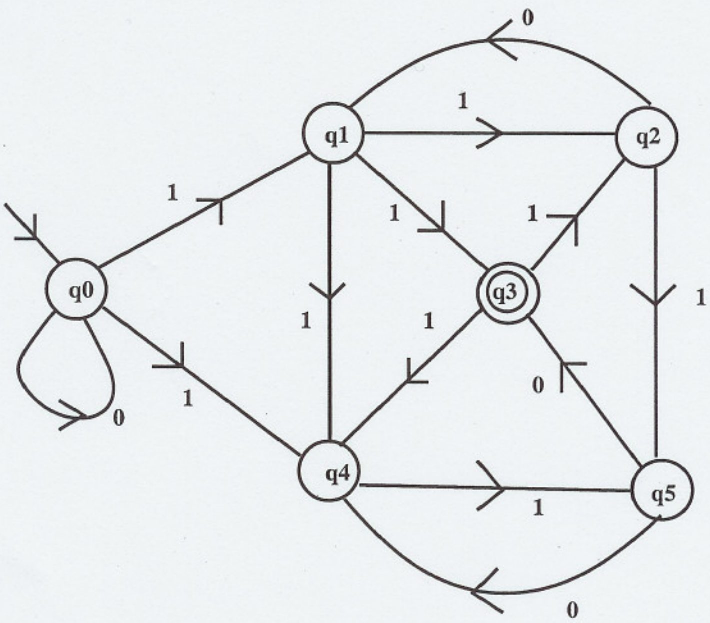
b. The language resulting from the **concatenation** of two **languages**. **(3 marks)**

c. The language **generated by a formal grammar** $G = (V, T, P, S)$. **(3 marks)**

d. A **null**–transition automaton, (also known as an ϵ-transition automaton). **(3 marks)**

e. The Church-Turing Hypothesis. **(3 marks)**

2. For the **non-deterministic** finite automaton below,



a. Describe its **state transition function** – $\delta : Q \times \{0, 1\} \rightarrow \mathcal{P}(Q)$. **(8 marks)**

b. Give the **full computation tree** of this automaton when it is given the word **01101** as input, indicating whether this word is accepted or not. **(7 marks)**

3. For the **right linear grammar**, $G = (V, \Sigma, S, P)$ in which $V = \{V_0, V_1, V_2, V_3\}$, $\Sigma = \{0, 1\}$, $S = V_0$, and P contains the production rules

$$\begin{array}{lcl} V_0 & \rightarrow & 1V_1 \\ V_1 & \rightarrow & 1 \mid\mid 0V_2 \mid\mid 1V_2 \mid\mid 1V_3 \\ V_2 & \rightarrow & 0V_1 \mid\mid 1V_1 \\ V_3 & \rightarrow & \epsilon \mid\mid 0V_1 \mid\mid 1V_1 \end{array}$$

a. Construct a (non-deterministic) finite automaton, $M = \langle Q, \Sigma, q_0, F, \delta \rangle$ for which a word $w \in \langle 0, 1 \rangle^*$ is accepted by M if and only if w can be generated by G. **(9 marks)**

b. Show that the word **100111** can be generated by G, giving the sequence of productions that are applied starting from $V_0 \rightarrow 1V_1$. **(6 marks)**

4. Give examples of each of the following:

a. A **context–free** language that is **not** a **regular** language. **(5 marks)**

b. A **recursive** language that is **not** a **context–free** language. **(5 marks)**

c. A **recursively enumerable** language that is **not** a **recursive** language. **(5 marks)**

Full credit requires a formal definition of a language with the relevant properties (but not a proof).

Section 2

Answer **two** questions in this section.

5. For the **context–free** grammar, $G = (V, \Sigma, S, P)$ with $V = \{A, B, C, S\}$; $\Sigma = \{a, b, c\}$; start symbol S and production rules, P, defined by

$$\begin{array}{lcl} S & \rightarrow & SBS \parallel aBd \parallel Ca \\ A & \rightarrow & BA \parallel AS \parallel ac \\ B & \rightarrow & BaA \parallel AC \parallel b \\ C & \rightarrow & CS \parallel ABCS \parallel c \end{array}$$

a. Identify all of the production rules of G that do not meet the conditions of **Chomsky Normal Form**. **(6 marks)**

b. Carefully show how G may be modified to give a context–free grammar $G' = (V', \Sigma, S, P')$ such that G' generates exactly the same language as G and for which every production rule in P' is in Chomsky Normal Form. (**9 marks**)

c. Suppose the production rules $C \rightarrow c$ and $B \rightarrow b$ in G are replaced by $C \rightarrow AB$ and $B \rightarrow Cb$. What property does the language generated by this new grammar have? Briefly justify your answer. (**5 marks**)

6. For the **deterministic** finite automaton $M = (Q, \Sigma, q_0, F, \delta)$ in which $Q = \{q_0, q_1, q_2, q_3, q_4, q_5\}$; $\Sigma = \{0, 1\}$; $F = \{q_4\}$ and $\delta : Q \times \Sigma \rightarrow Q$ is given by

q	σ	$\delta(q,\sigma)$	q	σ	$\delta(q,\sigma)$
q_0	0	q_1	q_0	1	q_5
q_1	0	q_2	q_1	1	q_3
q_2	0	q_1	q_2	1	q_5
q_3	0	q_5	q_3	1	q_4
q_4	0	q_5	q_4	1	q_3
q_5	0	q_5	q_5	1	q_5

a. Describe in detail how a **regular expression**, R is constructed from M with the property that the language – $L(R)$ – described by R is exactly the language $L(M)$ of words accepted by M. (**Hint:** Recall Arden's Rule: if R, S, and T are (regular) languages for which the relationship $R = S \cdot R \cup T$ holds and $\epsilon \notin S$, then $R = S^* \cdot T$ is the unique solution for R) (**15 marks**)

b. Why is the qualification $\epsilon \notin S$ significant in the statement of Arden's Rule? (**5 marks**)

7.

a. **State** the **Pumping Lemma for Context–Free Languages**. (**5 marks**)

b. Using the Pumping Lemma for Context–Free Languages, show that the language L_{MULT} over the alphabet $\{a, b, c\}$ defined by

$$L_{MULT} = \{ a^r b^s c^{r*s} : r, s \geq 1\}$$

is **not** a Context–Free language. (**10 marks**)

c. Suppose L_1 and L_2 are two languages over the same alphabet Σ and define the language L_{DIFF} to be

$$L_{DIFF} = \{ w \in \Sigma^* : w \in L_1 \text{ and } w \notin L_2\}$$

If L_1 is recursively enumerable but **not** recursive, and L_2 is recursive, is the language L_{DIFF} recursively enumerable? Briefly justify your answer. (**5 marks**)