PAPER CODE NO. COMP209

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THE UNIVERSITY
of LIVERPOOL

JANUARY 2003 EXAMINATIONS

Bachelor of Science : Year 2

AUTOMATA AND FORMAL LANGUAGES

TIME ALLOWED : Two Hours

INSTRUCTIONS TO CANDIDATES

Section 1: Answer **ALL** questions
Section 2: Answer any **TWO** questions

If you attempt to answer more than the required number of questions (in any section), the marks awarded for the excess questions will be discarded (starting with your lowest mark).

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Section 1

Answer **all** questions in this section.

1. Define what is meant by the following terms,

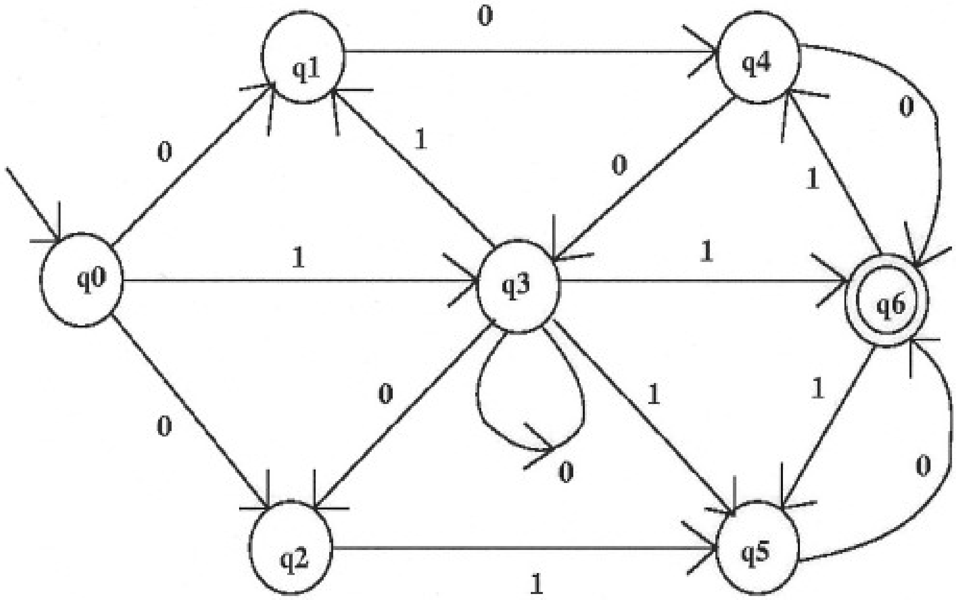
   (a) A **language** over an alphabet, Σ. **(3 marks)**
   (b) The **empty word**, ε. **(3 marks)**
   (c) The language formed by **concatenating** two languages. **(3 marks)**
   (d) The **Church-Turing Hypothesis**. **(3 marks)**
   (e) **Right-Linear Grammar**. **(3 marks)**

2. For the **non-deterministic Finite Automaton**,

$$M = (Q, \Sigma, q_0, F, \delta)$$

shown below, and in which

$Q=\{q_0, q_1, q_2, q_3, q_4, q_5, q_6\}$; $\Sigma=\{0,1\}$; $F=\{q_6\}$



(a) Give the state-transition function, $\delta : Q \times \{0,1\} \rightarrow \wp(Q)$. **(7 marks)**
(b) Give the tree of possible state sequences that could occur when M reads the word **110010**. **(8 marks)**

3. Let *EVEN-0* be the language consisting of all **even length** words that start and end with a single symbol **1**, contain only **0** symbols in between, and have length at least 4, i.e.

$$EVEN\text{-}0 = \{\mathbf{1}\cdot\mathbf{0}^{2n}\cdot\mathbf{1} : \text{for some } n\geq 1\}$$

Consider the **six regular expressions** (1)-(6) below:

(1) $\mathbf{(0+1)^{*}\cdot 1\cdot 0\cdot 0\cdot (0\cdot 0)^{*}\cdot 1\cdot (0+1)^{*}}$

(2) $\mathbf{1\cdot (0\cdot 0\cdot (0\cdot 0)^{*})\cdot 1}$

(3) $\mathbf{(0+1\cdot 1+1\cdot 0\cdot (0\cdot 0)^{*}1+1\cdot 0\cdot 0\cdot (0\cdot 0)^{*}\cdot 1\cdot (0+1))\cdot (0+1)^{*}}$

(4) $\mathbf{(1^{*}\cdot 0\cdot 1^{*}\cdot 0)^{*}}$

(5) $\mathbf{1\cdot 0\cdot 0\cdot (0\cdot 0)^{*}\cdot 1\cdot (0+1)^{*}}$

(6) $\mathbf{(0+1)^{*}\cdot 1\cdot 0\cdot 0\cdot (0\cdot 0)^{*}\cdot 1}$

For each of the sets of words over the alphabet {**0**,**1**} below, **state** which of these six regular expressions describes it. Give a brief justification of your answer in each case.

(a) The set of words that are members of *EVEN-0*. **(3 marks)**
(b) The set of words that **start** with some word *w* in *EVEN-0* followed by any sequence of the symbols **0** and **1**. **(4 marks)**
(c) The set of words that contain **at least one sub-word**, *w* in *EVEN-0*. **(4 marks)**
(d) The set of **non-empty** words that are **not** members of *EVEN-0*. **(4 marks)**

4. Give **one** example of,
   (a) A **Context-Free** language that is **not** a regular language. **(5 marks)**
   (b) A **recursive** language that is **not** a Context-Free language. **(5 marks)**
   (c) A **recursively enumerable** language that is **not** a recursive language. **(5 marks)**

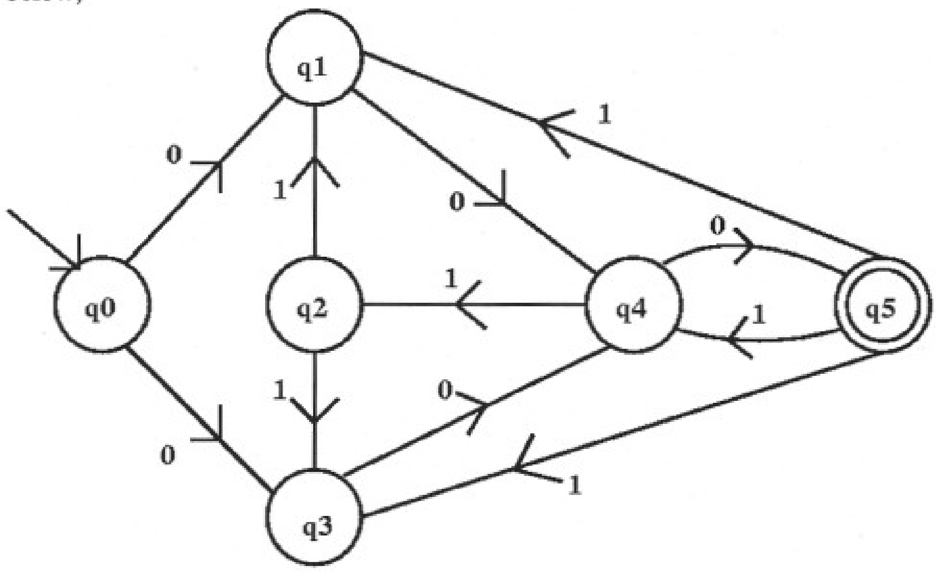
Section 2

Answer **two** questions from this section.

5. For the **non-deterministic** finite automaton,

$$M_{nd} = (Q_{nd}, \{0,1\}, q_0, F_{nd}, \delta_{nd})$$

with $Q_{nd}=\{q_0, q_1, q_2, q_3, q_4, q_5\}$, $F_{nd}=\{q_5\}$, and δ_{nd} as defined by the diagram below,



(a) Construct the equivalent **deterministic** automaton

$$M_d = (Q_d, \{0,1\}, q_0, F_d, \delta)$$

Q_d should have **ten** states. You may describe δ using either a diagram or in tabular form. (**15 marks**)

(b) Are **both** the states q_1 and q_3 necessary? Briefly justify your answer. (**5 marks**)

6. For the Context-Free grammar, $G=(V,\Sigma,P,S)$ in which $V=\{S,X,Y,Z\}$, $\Sigma=\{\mathbf{a},\mathbf{b},\mathbf{c}\}$ and with production rules, P, given by

$$\begin{aligned} S &\rightarrow XY\mathbf{a} \mid S\mathbf{b}Z \\ X &\rightarrow \mathbf{c}S \mid YZ \\ Y &\rightarrow ZYS \mid XSX \\ Z &\rightarrow SX \mid Y\mathbf{a}X \\ X &\rightarrow \mathbf{a} \\ Y &\rightarrow \mathbf{b} \\ Z &\rightarrow \mathbf{c} \end{aligned}$$

(a) Identify **all** of the production rules of G that are not in **Chomsky Normal Form. (6 marks)**

(b) Carefully describe how G should be modified to a Context-Free grammar, G_C, such that G_C is in Chomsky Normal Form **and** generates exactly the same language as G. **(9 marks)**

(c) If L_1 and L_2 are two Context-Free languages, briefly explain why the language formed by **concatenating** L_1 and L_2 is also a Context-Free language. **(5 marks)**

7.

(a) **State** the **Pumping Lemma for Context-Free Languages. (5 marks)**

(b) Show that the language L_{div} over the alphabet $\{\mathbf{a}, \mathbf{b}, \mathbf{c}\}$,

$$L_{div} = \{\ \mathbf{a}^x\ \mathbf{b}^y\ \mathbf{c}^{x/y} : x, y \geq 1, y \text{ is an exact divisor of } x\}$$

is **not** a Context-Free language, by applying the Pumping Lemma for Context-Free languages to some word of the form $\mathbf{a}^{m*m}\mathbf{b}^m\mathbf{c}^m$. **(10 marks)**

(c) Suppose $\eta(M)$ is an encoding of Turing machine programs. Is the following language **recursive** or **recursively enumerable**?

$\{\ \eta(M)\#q_k : q_k$ is a state of M that is reached when M is given the empty word as its input$\}$

Briefly justify your answer. **(5 marks)**

END OF PAPER