PAPER CODE NO. COMP209

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THE UNIVERSITY
of LIVERPOOL

JANUARY 2002 EXAMINATIONS

Bachelor of Science : Year 2

AUTOMATA AND FORMAL LANGUAGES

TIME ALLOWED : Two Hours

INSTRUCTIONS TO CANDIDATES

Section 1: Answer **all** questions

Section 2: Answer **2** questions

If you attempt to answer more than the required number of questions (in any section), the marks awarded for the excess questions will be discarded (starting with your lowest mark).

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Section 1

Answer **all** questions in this section.

1. Define what is meant by the following terms,

(a) A **language** over an alphabet, Σ. (**3 marks**)

(b) The **empty word**, ε. (**3 marks**)

(c) The **empty language**. (**3 marks**)

(d) The **language generated by** a Formal Grammar $G = (V, T, P, S)$. (**3 marks**).

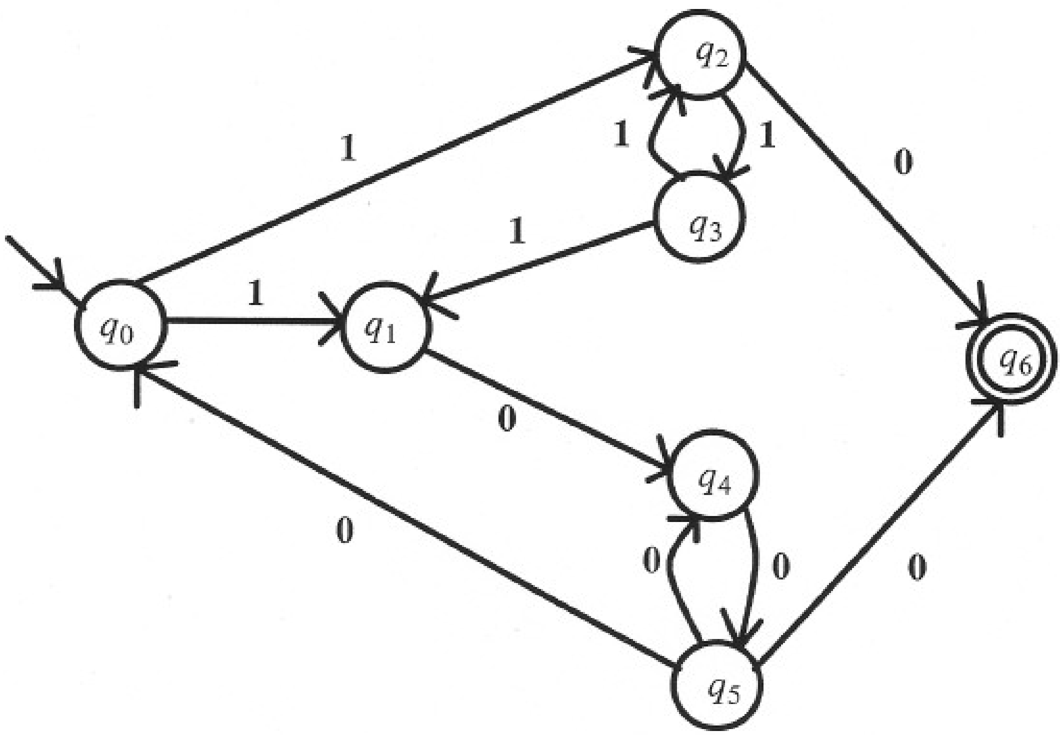
(e) The Church-Turing Hypothesis. (**3 marks**)

2. For the non-deterministic Finite Automaton,

$$M = (Q, \Sigma, q_0, F, \delta)$$

shown below, and in which

$$Q = \{ q_0, q_1, q_2, q_3, q_4, q_5, q_6 \} \quad ; \quad \Sigma = \{ 0, 1 \} \quad ; \quad F = \{ q_6 \}$$



(a) Give the state-transition function,

$$\delta : Q \times \{ 0, 1 \} \rightarrow \wp(Q).$$ (**7 marks**)

(b) Give the tree of possible state sequences that could occur when M reads the word 111000. (**8 marks**)

3. Consider the Right-Linear Grammar (*RLG*)

$$G = (V, \Sigma, P, S)$$

in which,

$$V = \{ V_0, V_1, V_2, V_3 \} \quad ; \quad \Sigma = \{ 0, 1 \} \quad ; \quad S = V_0$$

and with production rules, P, given by

$$\begin{array}{lcl} V_0 & \rightarrow & 1 \mid 0\,V_1 \mid 1\,V_3 \\ V_1 & \rightarrow & 0 \mid 0\,V_3 \mid 1\,V_1 \\ V_2 & \rightarrow & 0\,V_2 \mid 1\,V_1 \\ V_3 & \rightarrow & 0\,V_0 \mid 1\,V_2 \end{array}$$

For each of the words below show whether the word can be generated by G.

a) 001110 (**6 marks**)

b) 110001 (**6 marks**)

c) Give a **brief** justification as to why no word that **ends** with 11 is generated by G. (**3 marks**)

4. Give **one** example of,

(a) A Context-Free language that is **not** a regular language. (**5 marks**)

(b) A **recursive** language that is **not** a Context-Free language. (**5 marks**)

(c) A **recursively enumerable** language that is **not** a recursive language (**5 marks**)

[It is not necesssary to **prove** that your examples have the property required.]

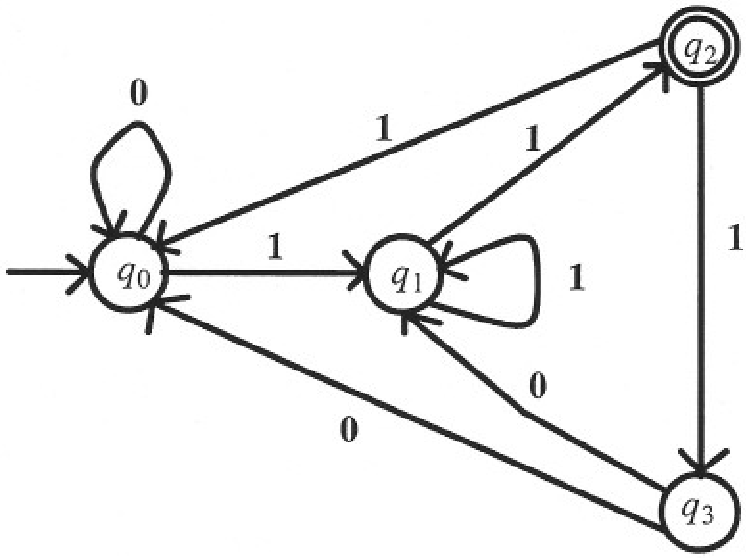
Section 2

Answer **two** of Questions 5, 6, and 7.

5. For the **non-deterministic** finite automaton,

$$M_{nd} = (Q_{nd}, \{ 0,1 \}, q_0, F_{nd}, \delta_{nd})$$

with $Q_{nd} = \{ q_0, q_1, q_2, q_3 \}$, $F_{nd} = \{ q_2 \}$, and δ_{nd} as defined by the diagram below,



(a) Construct the equivalent **deterministic** automaton,

$$M_d = (Q_d, \{ 0,1 \}, q_0, F_d, \delta)$$

Q_d should have **six** states. You may describe δ using either a diagram or in tabular form. (**15 marks**)

(b) The class of regular languages is closed under **finite union**, i.e. for any constant value k, if

$$L_1, L_2, \cdots, L_k$$

are regular languages, then $\bigcup_{i=1}^{k} L_i$ is also a regular language.

Is the class of regular languages closed under **infinite** union, i.e. if

$$L_1, L_2, L_3, \ldots, L_k, \ldots,$$

is some **infinite** collection of regular languages, is the language $\bigcup_{i=1}^{\infty} L_i$ always a regular language? Justify your answer. (**5 marks**)

6.

For the Context-Free grammar, $G = (V, \Sigma, P, S)$ in which, $V = \{ S, A, C \}$, $\Sigma = \{ a, b, c, d \}$ and with production rules, P, given by

$$S \rightarrow A\,C\,A \mid S \rightarrow C\,A\,C$$
$$A \rightarrow a\,A \mid A \rightarrow A\,b$$
$$C \rightarrow c\,C\,c \mid C \rightarrow d\,C\,d$$
$$A \rightarrow a \mid A \rightarrow b \mid C \rightarrow c \mid C \rightarrow d$$

(a) Identify **all** of the production rules of G that violate **Chomsky Normal Form** for Context-Free Grammars. (**6 marks**)

(b) Carefully describe how G should be modified to a Context-Free grammar, G_C, such that G_C is in Chomsky Normal Form **and** generates exactly the same language as G. (**8 marks**)

(c) Give **two** reasons why the algorithm of Cocke, Younger, and Kasami (the *CYK-algorithm*) for testing whether a word w can be generated by a Context-free Grammar $G = (V, \Sigma, P, S)$ is unsuitable as a method for performing syntax checking for programming languages defined using a Context-Free Grammar. (**6 marks**).

7.

(a) **State** the **Pumping Lemma for Context-Free languages**. (**4 marks**).

(b) Show that the language over $\{ a, b, c \}$,

$$L_{mult} = \{ a^x \, b^y \, c^{xy} : x, y \geq 1 \}$$

(xy denoting x multiplied by y),

is not a Context-Free language, by applying the Pumping Lemma for Context-Free languages to some word of the form $a^m \, b^m \, c^{m^2} \in L_{mult}$. (**10 marks**)

(c) Assuming a recursive encoding function, $\eta(M)$ of Turing Machine programs, state (giving a **brief** justification of your answer) whether the following languages are **recursive**:

i. $\{ \eta(M) : M$ **halts** on the empty word $\}$ (**3 marks**)

ii. $\{ \eta(M) : M$ makes **exactly** $|\eta(M)|$ moves on the empty word $\}$ (**3 marks**)

END OF PAPER