PAPER CODE NO.	EXAMINER : **Dr Paul E Dunne**
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THE UNIVERSITY
of LIVERPOOL

JANUARY 2006 EXAMINATIONS

Bachelor of Science: Foundation Year
Bachelor of Science: Year 1
Bachelor of Science: Year 2
No qualification aimed for: Year 1

DECISION, COMPUTATION AND LANGUAGE

TIME ALLOWED: Two Hours

INSTRUCTIONS TO CANDIDATES

Answer **all** questions in Section 1 and **two** questions from Section 2.
Section 1 is worth 60% of the marks and Section 2 is worth 40% of the marks.

If you attempt to answer more than the required number of questions (in any section), the marks awarded for the excess questions will be discarded (starting with your lowest mark).

Section 1

Answer **all** questions in this section.

1. Define what is meant by the following terms,

a. A **word** over an alphabet, Σ. **(3 marks)**

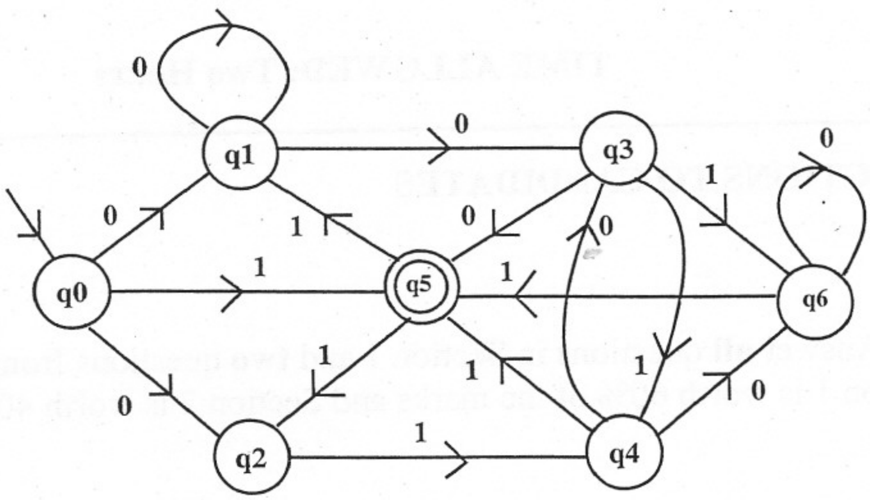
b. The **language** resulting from the **concatenation** of two **languages. (3 marks)**

c. The language **accepted by a non-deterministic finite automaton** $M = (Q, \Sigma, q_0, F, \delta)$. **(3 marks)**

d. A **context-free grammar. (3 marks)**

e. The Church-Turing Hypothesis. **(3 marks)**

2. For the **non-deterministic** finite automaton below,



a. Describe its **state transition function** – $\delta : Q \times \{0, 1\} \rightarrow \mathcal{P}(Q)$. **(8 marks)**

b. Give the **full computation tree** of this automaton when it is given the word **010101** as input, indicating whether this word is accepted or not. **(7 marks)**

3. For the **deterministic**, finite automaton $M = (Q, \{0,1\}, q_0, F, \delta)$ in which $Q = \{q_0, q_1, q_2, q_3, q_4\}$, $F = \{q_3, q_4\}$ and $\delta : Q \times \{0,1\} \rightarrow Q$ is given by

q	σ	$\delta(q,\sigma)$	q	σ	$\delta(q,\sigma)$
q_0	0	q_1	q_0	1	q_2
q_1	0	q_2	q_1	1	q_3
q_2	0	q_4	q_2	1	q_1
q_3	0	q_4	q_3	1	q_0
q_4	0	q_0	q_4	1	q_3

a. Construct a **right-linear grammar**, $G = \langle V, T, S, P \rangle$ such that G generates a word w if and only if w is accepted by M. **(9 marks)**

b. Give the sequence of productions that are applied in this grammar to generate the word **011010**. **(6 marks)**

4. Give examples of each of the following:

a. A **context–free** language that is **not** a **regular** language. **(5 marks)**

b. A **recursive** language that is **not** a **context–free** language. **(5 marks)**

c. A **recursively enumerable** language that is **not** a **recursive** language. **(5 marks)**

Full credit requires a formal definition of a language with the relevant properties (but not a proof).

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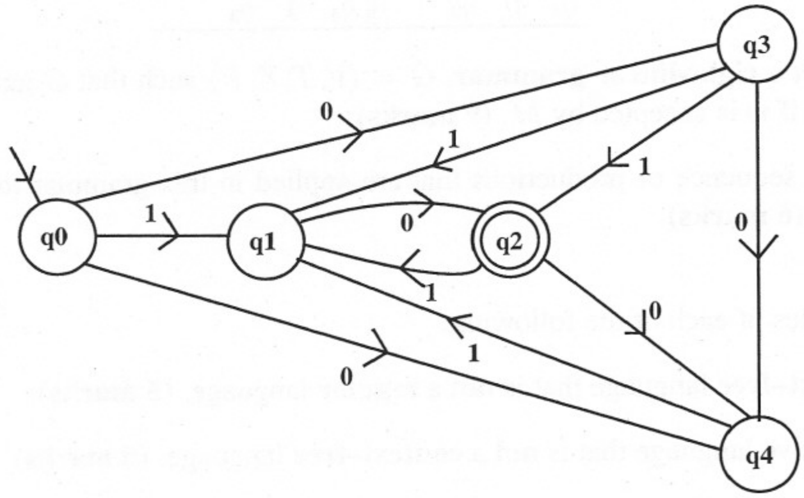
Section 2

Answer **two** questions in this section.

5.

a. Carefully describe the construction of a **deterministic** finite automaton equivalent to the **non-deterministic** finite automaton shown below **(12 marks)**



b. Consider the language, $L_{SUBTRACT}$ over $\{0, 1\}$, defined by the following set of words,

$$L_{SUBTRACT} = \{ 0^n \cdot 1^m \cdot 0^{n-m} : n > m \geq 0 \}$$

Using the **Pumping Lemma for Regular Languages**, prove that $L_{SUBTRACT}$ is not a regular language. (**8 marks**)

6.

a. For the **deterministic** finite automaton $M = (Q, \Sigma, q_0, F, \delta)$ with $Q = \{q_0, q_1, q_2, q_3, q_4, q_5\}$; $\Sigma = \{0, 1\}$, $F = \{q_4\}$ and δ given by

q	σ	$\delta(q,\sigma)$	q	σ	$\delta(q,\sigma)$
q_0	0	q_1	q_0	1	q_2
q_1	0	q_0	q_1	1	q_3
q_2	0	q_5	q_2	1	q_0
q_3	0	q_4	q_3	1	q_5
q_4	0	q_3	q_4	1	q_5
q_5	0	q_5	q_5	1	q_5

Show in detail how a **regular expression**, R_M, is constructed with the property that a word is accepted by M if and only if it belongs to the language described by the regular expression R_M.
[**Hint:** Use Arden's Rule – if R, S, T, are regular languages for which $\epsilon \notin S$ and $R = S \cdot R + T$, then $R = S^* \cdot T$] (**12 marks**)

b. Suppose $\Sigma = \{\sigma_1, \ldots, \sigma_k\}$ is an alphabet of k symbols and $L \subseteq \Sigma^*$ is a language over Σ. Consider the language $bin(L)$ over the alphabet $\{0, 1\}$ defined as

$$bin(L) \quad = \quad \{\ 0^{i_1}10^{i_2}1 \cdots 10^{i_r}\ :\ \sigma_{i_1}\sigma_{i_2}\ \cdots\ \sigma_{i_r}\ \in\ L\}$$

If L is a **regular** language then is it the case that $bin(L)$ is also a regular language? Give a brief justification for your answer. **(8 marks)**

7. For the **context–free** grammar, $G = (V, \Sigma, S, P)$ with $V = \{A, B, C, S\}$; $\Sigma = \{a, b, c, d\}$; start symbol S and production rules, P, defined by

$$\begin{array}{lcl} S & \rightarrow & SBc \ \|\ BAC \ \|\ CA \\ A & \rightarrow & CS \ \|\ ABC \ \|\ ad \\ B & \rightarrow & CaS \ \|\ BC \ \|\ b \\ C & \rightarrow & AB \ \|\ SCAS \ \|\ c \end{array}$$

a. Identify, giving reasons for each case, all of the production rules of G that do not meet the conditions of **Chomsky Normal Form**, **(6 marks)**

b. Carefully show how G may be modified to give a context–free grammar $G' = (V', \Sigma, S, P')$ such that G' generates exactly the same language as G and for which every production rule in P' is in Chomsky Normal Form. **(6 marks)**

c. Consider the language, $L^k_{CAN-HALT}$ of **Turing Machine** properties given by

$$L^k_{CAN-HALT} \quad = \quad \{\ \langle M, k\rangle\ :\ M \textbf{ halts} \text{ on } \textbf{at least one} \text{ word in } \{0,1\}^k\}$$

Show that $L_{CAN-HALT}$ is **recursively enumerable** (r.e.). **(8 marks)**