PAPER CODE NO.	EXAMINER : Leszek Gąsieniec
COMP202	DEPARTMENT : Computer Science Tel. No. 7943686

THE UNIVERSITY
of LIVERPOOL

May 2004 EXAMINATIONS

Bachelor of Science : Year 2

Complexity of Algorithms

TIME ALLOWED : 2 hours

INSTRUCTIONS TO CANDIDATES

Answer **four** questions only.

If you attempt to answer more questions than the required number of questions (in any section), the marks awarded for the excess questions will be discarded (starting with your lowest mark).

Question 1

1.A State the definition of *AVL trees* and explain why the tree T shown below is not an AVL tree. **[5 marks]**

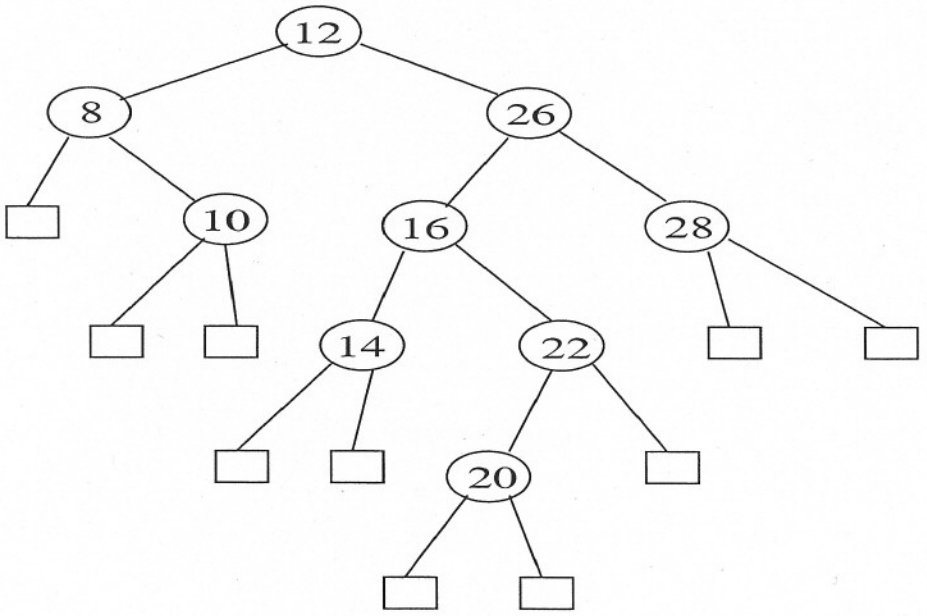

Figure 1: Tree T

Assume that **20** is the last inserted element. Propose a rotation transforming this tree into a proper AVL tree and draw the structure of the tree after the transformation. **[10 marks]**

1.B Explain what the *one-time pad* cipher is. List also two advantages and two disadvantages of the *one-time pad* cipher. **[5 marks]**

1.C What is the *Bellman-Ford algorithm* used for and what is its worst-case time complexity? **[5 marks]**

Question 2

2.A What is the main observation used in Euclid's algorithm? Trace the execution of Euclid's algorithm on input: 147 and 91.

[10 marks]

2.B State the definition of the *biconnectivity property* in graphs. Explain why graph G is not biconnected and propose a new edge, s.t., after its insertion to G the graph becomes biconnected.

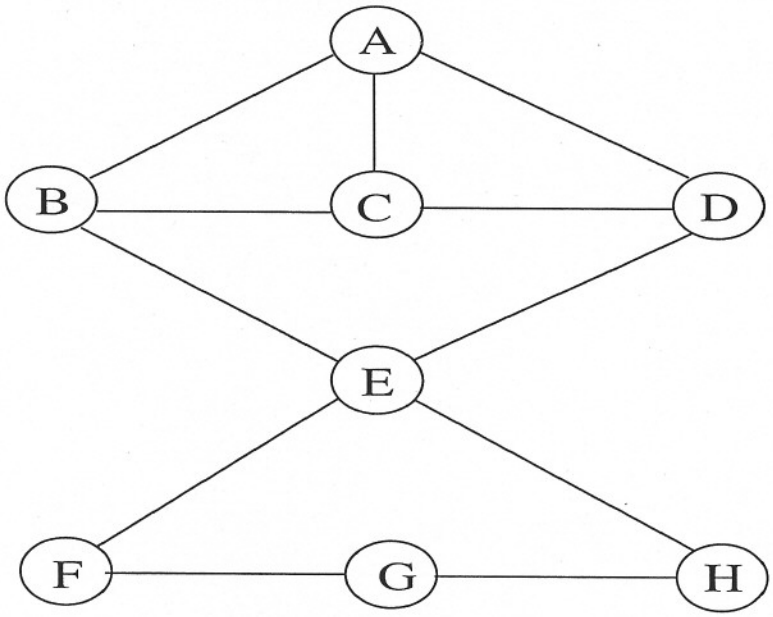

Figure 2: Graph G

[10 marks]

2.C Explain the difference between two important models in distributed algorithm design: the *synchronous model* and the *asynchronous model.* **[5 marks]**

Question 3

3.A Let T be a binary tree with keys being positive integers. Write pseudocode of a recursive procedure that decides whether the sum of odd keys is larger than the sum of even keys stored in T. What is the time complexity of your solution?

[15 marks]

3.B Explain the difference between a *decision problem* and an *optimisation problem*. Explain also how we can turn an optimisation problem into a decision problem.

[5 marks]

3.C If L is a language in NP, why does this not necessarily mean that the complement of L (co-L) belongs to NP?

[5 marks]

Question 4

4.A Insert elements of a sequence $S = \{13, 9, 11, 8, 7, 3, 5\}$ into (initially) an empty *Heap* H. The elements are inserted one by one, in order of their appearance in S. Draw the tree representation of H after insertion of each element. Finally draw the vector representation of H after the last insertion took place.

[15 marks]

4.B Explain the main difference between the *divide-and-conquer* and *decrease-and-conquer* methods. Give names of two algorithms based on the divide-and-conquer method and names of two algorithms based on the decrease-and-conquer method.

[5 marks]

4.C Explain the main differences between two basic data structures: *stack* and *queue*.

[5 marks]

Question 5

5.A Solve the *leader election problem* in a directed ring R, see Figure 3, perform-

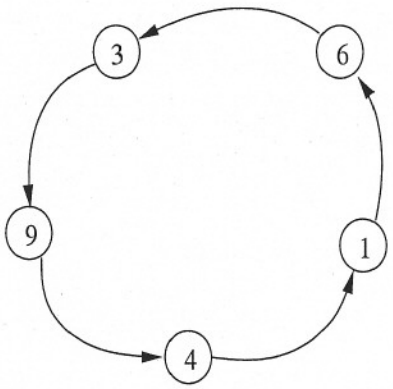

Figure 3: Directed Ring R

ing the *RingLeader* algorithm. Draw the content of all messages that are sent during each consecutive round. (Use short description of messages, i.e., represent a message "Candidate is i" by "C:i" and a message "Leader is j" by "L:j".) What is the exact number of messages that have been sent by the *RingLeader* algorithm in this particular case? **[15 marks]**

5.B State the definition of the *vertex-cover* problem in (undirected) graph $G = (V, E)$. Which other problem can be reduced to the vertex-cover problem in order to prove that the vertex-cover problem is NP-hard? Comment briefly on the reduction process. **[7 marks]**

5.C What is the time complexity of the *DFS traversal* in a graph with n vertices and m edges? List also two other graph problems that can be solved with the help of the *DFS traversal.* **[3 marks]**