PAPER CODE NO.	EXAMINER : Leszek Gasieniec
COMP 202	DEPARTMENT : Computer Science Tel. No. 43686

THE UNIVERSITY
of LIVERPOOL

MAY 2003 EXAMINATIONS

Bachelor of Arts : Year 2
Bachelor of Science : Year 2
Master of Science: Year 1
No qualification aimed for: Year 1

COMPLEXITY OF ALGORITHMS

TIME ALLOWED : Two Hours

INSTRUCTIONS TO CANDIDATES

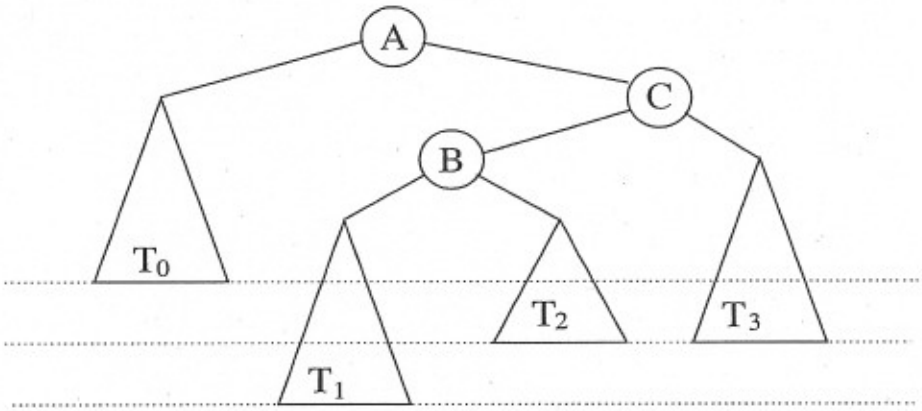
Candidates will be assessed on their best four answers.

If you attempt to answer more than the required number of questions (in any section), the marks awarded for the excess questions will be discarded (starting with your lowest mark).

Question 1

1.A State the definition of *AVL trees* and explain how an AVL tree T shown below will look after a *double rotation* is applied at its root A.

AVL tree T



[10 marks]

1.B Explain what the *substitution cipher* is. State also the definition of a *Caesar cipher* and comment on how it can be broken. **[10 marks]**

1.C What is the *Bellman-Ford algorithm* used for and what is its worst-case time complexity? **[5 marks]**

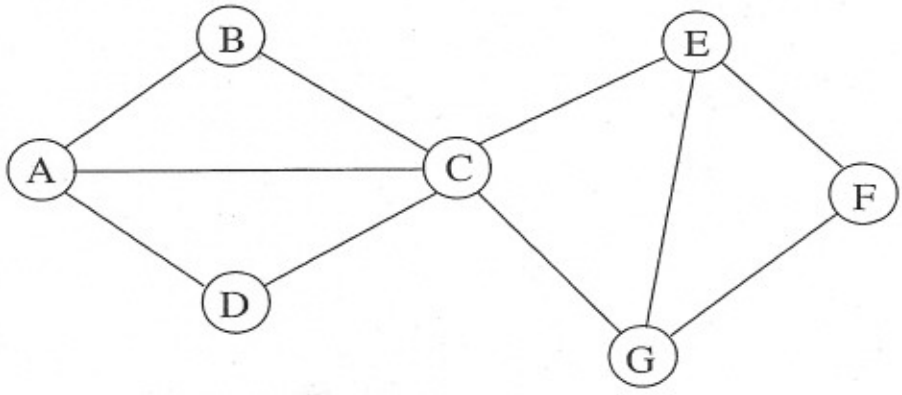
Question 2

2.A What is the main observation used in Euclid's algorithm? Trace the execution of Euclid's algorithm on the input: 87 and 63.

[10 marks]

2.B State the definition of the *biconnectivity property* in graphs. Explain why graph G is not biconnected and propose an additional edge that would make this graph biconnected.

Graph G



[10 marks]

2.C Explain the difference between two models in distributed algorithm design: *synchronous model* and *asynchronous model.* **[5 marks]**

Question 3

3.A Write a pseudocode of a recursive procedure that computes the sum of all keys stored in a binary tree T. What type of tree traversal is used by this procedure? What is the time complexity of your solution? **[15 marks]**

2.B Explain the difference between the *divide-and-conquer* and *decrease-and-conquer* methods. Give names of two algorithms based on the divide-and-conquer method and names of two algorithms based on the decrease-and-conquer method. **[7 marks]**

3.C What is the meaning of *PTAS*? **[3 marks]**

Question 4

4.A Insert the elements of a sequence $S = \{11, 8, 9, 7, 6, 3, 5\}$ into initially empty *Heap* H. The elements are inserted one by one, in order of an appearance in S. Draw a tree representation of H after insertion of each element. Finally draw a vector representation of H after the last insertion took place.

[15 marks]

4.B Explain the difference between a *decision problem* and a *optimisation problem*. Explain also how we can turn an optimisation problem into a decision problem.

[5 marks]

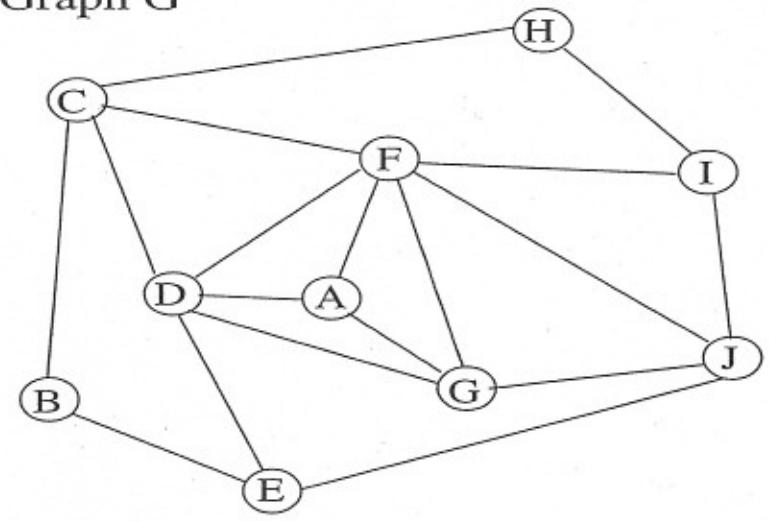
4.C What is the difference between *worst-case* and *average-case* time complexity?

[5 marks]

Question 5

5.A Draw a *BFS spanning tree* rooted in vertex A in the following graph G'. Explain which data structure is very useful in construction of a BFS tree.

Graph G'



[15 marks]

5.B State the definition of a *vertex-cover* problem in (undirected) graph $G = (V, E)$. Which other problem can be reduced to the vertex-cover problem in order to prove that vertex-cover problem is NP-hard? Comment on the reduction process shortly. **[7 marks]**

5.C State the definition of a *leader election* problem. **[3 marks]**