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UNIVERSITY OF LIVERPOOL

May 2007 EXAMINATIONS

Bachelor of Arts : Year 2
Bachelor of Science : Year 2
No qualification aimed for : Year 1

Complexity of Algorithms

TIME ALLOWED : Two hours

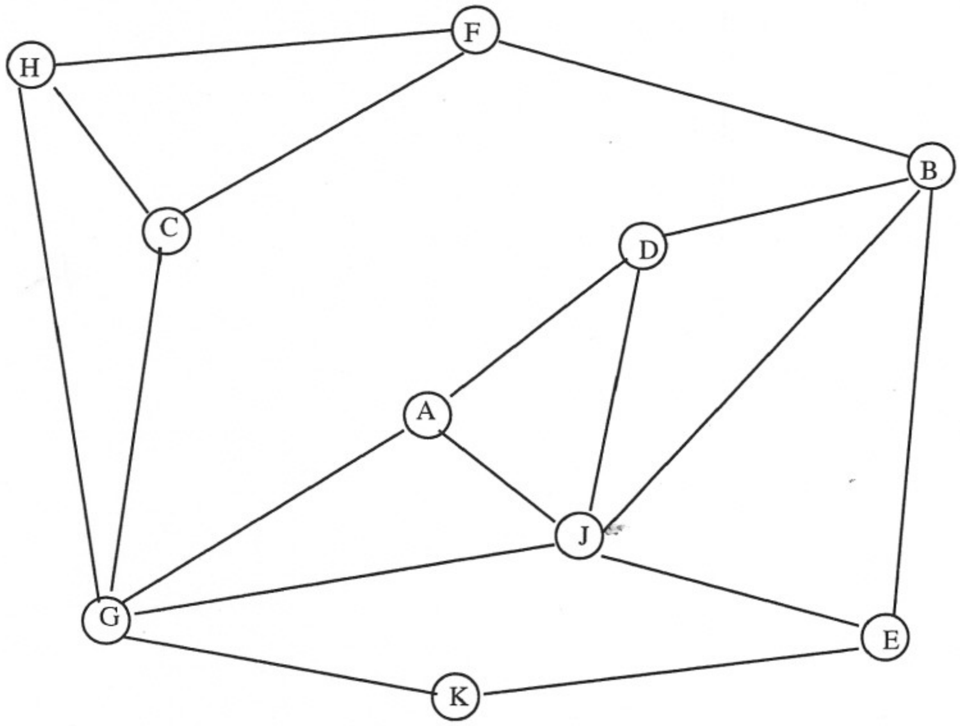
INSTRUCTIONS TO CANDIDATES

Answer **FIVE** questions.

If you attempt to answer more questions than the required number of questions (in any section), the marks awarded for the excess questions answered will be discarded (starting with your lowest mark).

(1) (a) Draw a BFS (breadth-first search) spanning tree rooted at A in the following graph. Write down a list of vertices of the graph in the order in which you visit them during your search (starting with vertex A).

What standard kind of data structure is very useful in constructing such a BFS spanning tree? **[10 marks]**



(b) Explain the basic mechanism behind the MergeSort and QuickSort algorithms. Be sure to point out what is similar about them, and how they differ. (DO NOT give pseudo-code, just a three to four line high level description of each procedure should suffice.) **[10 marks]**

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(2) **(a)** Let T be a binary tree with depth d. What is the maximum number of *external nodes (leaves)* that T can have? With depth d, what is the maximum *total* number of nodes that T can have? **[10 marks]**

(b) A $\{0,1\}$ **Integer Programming** problem is one that can be written in the form

$$\max \sum_{i=1}^{n} \alpha_i \cdot x_i$$

subject to

$$x_i \in \{0,1\} \text{ for all } i$$

$$\begin{array}{rcl} a_1 \cdot x_1 + a_2 \cdot x_2 + \cdots + a_n \cdot x_n & \leq & A \\ b_1 \cdot x_1 + b_2 \cdot x_2 + \cdots + b_n \cdot x_n & \leq & B \\ \vdots \quad\quad \vdots \quad\quad \vdots \quad\quad \vdots & & \vdots \\ t_1 \cdot x_1 + t_2 \cdot x_2 + \cdots + t_n \cdot x_n & \leq & T \end{array}$$

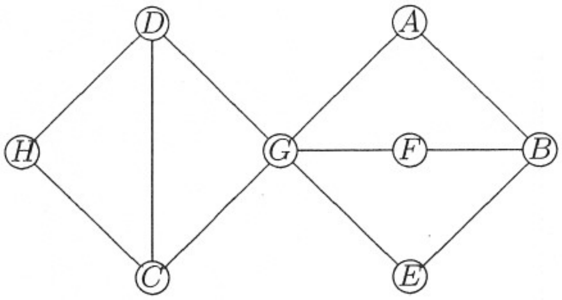
where all of the coefficients $\alpha_i, a_i, b_i, \ldots, t_i$ are rational numbers (or integers).

What is the question that the *decision version* of $\{0,1\}$ Integer Programming would be asking?
Explain *why* the decision version of this problem is in the class $\mathcal{NP}$.

[10 marks]

(3) (a) State the definition of the **biconnectivity** property in graphs. Explain why the graph given below is not biconnected. Then propose an edge whose addition to the graph will make it biconnected.

[10 marks]



(b) Define the **Fractional Knapsack Problem** and the $\{0, 1\}$ **Knapsack Problem**, explaining carefully how they differ.

What is the name of the general solution method that we can use to solve the Fractional Knapsack Problem, and what is the *different* solution method that we can use to solve the $\{0, 1\}$ Knapsack Problem?

[10 marks]

(4) (a) Explain what a *substitution cipher* is. What is a *Caesar cipher*? Comment on how a Caesar cipher can be broken. **[10 marks]**

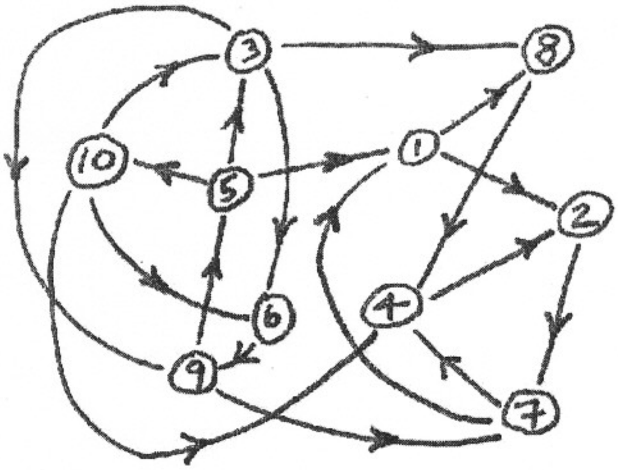
(b) What is the main observation used in Euclid's algorithm? Trace Euclid's algorithm on the inputs 408 and 222. **[10 marks]**

(5) (a) Insert the elements of the sequence $S = \{11, 8, 10, 4, 5, 2, 7\}$ into an initially empty heap H (that maintains the minimum at the root). The elements are inserted one at a time in the order of their appearance in H. Draw a tree representation of H after the insertion of each element. Finally give a vector representation of H after the last insertion took place. **[10 marks]**

(b) State necessary and sufficient conditions for a connected undirected graph $G = (V, E)$ to have an Eulerian tour.

Give an example of a graph that has a Hamilton cycle but does not have an Eulerian tour. **[10 marks]**

(6) **(a)** Consider the directed graph below.



For each set of vertices given below, state whether or not they are in the same strongly connected component, giving a brief justification for your answer.

(i) Vertices 7 and 8. **[3 marks]**

(ii) Vertices 9 and 10. **[3 marks]**

(iii) Vertices 8, 9, and 10. **[4 marks]**

(b) Clearly state the definition of the following $\mathcal{NP}$-complete problems giving the form that an **instance** of the problem has, as well as the **question** being asked for such an instance.

(i) The **Subset Sum Problem**. **[5 marks]**

(ii) The 3-**Satisfiability Problem** (3-SAT). **[5 marks]**

END OF PAPER