PAPER CODE NO. **COMP202**	EXAMINER : Russell Martin DEPARTMENT : Computer Science Tel. No. 7943695

THE UNIVERSITY
of LIVERPOOL

May 2006 EXAMINATIONS

Bachelor of Arts : Year 2
Bachelor of Science : Year 2
No qualification aimed for : Year 1

Complexity of Algorithms

TIME ALLOWED : 2 hours

INSTRUCTIONS TO CANDIDATES

Answer **five** of the six questions.

If you attempt to answer more questions than the required number of questions (in any section), the marks awarded for the excess questions will be discarded (starting with your lowest mark).

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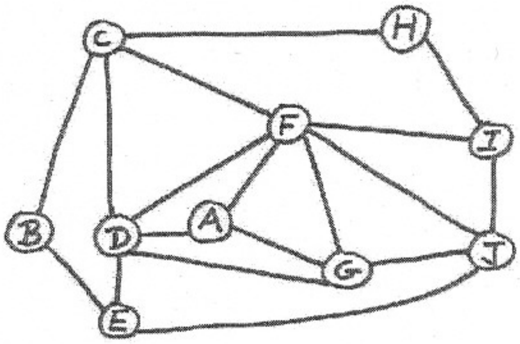
(1) (a) Draw a BFS (breadth-first search) spanning tree rooted at A in the following graph. Write down a list of vertices of the graph in the order in which you visit them during your search (starting with vertex A).

What standard kind of data structure is very useful in constructing such a BFS spanning tree? **[15 marks]**



(b) Explain the basic mechanism behind the quicksort algorithm. (DO NOT give pseudo-code, just a three to four line high level description of the procedure should suffice.) **[5 marks]**

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(2) (a) Let T be a binary tree with depth d. What is the maximum number of *external nodes (leaves)* that T can have? With depth d, what is the maximum *total* number of nodes that T can have? **[10 marks]**

(b) A $\{0,1\}$ **Integer Programming** problem is one that can be written in the form

$$\max \sum_{i=1}^{n} \alpha_i \cdot x_i$$

subject to

$$x_i \in \{0,1\} \text{ for all } i$$

$$\begin{array}{rcl} a_1 \cdot x_1 + a_2 \cdot x_2 + \cdots + a_n \cdot x_n & \leq & A \\ b_1 \cdot x_1 + b_2 \cdot x_2 + \cdots + b_n \cdot x_n & \leq & B \\ \vdots \quad \vdots \quad \vdots \quad \vdots & & \vdots \\ t_1 \cdot x_1 + t_2 \cdot x_2 + \cdots + t_n \cdot x_n & \leq & T \end{array}$$

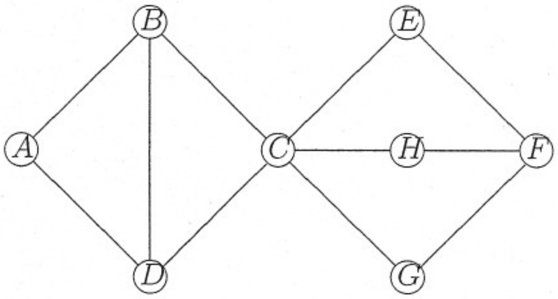
where all of the coefficients $\alpha_i, a_i, b_i, \ldots, t_i$ are positive rational numbers (or positive integers).

What is the question that the *decision version* of $\{0,1\}$ Integer Programming would be asking?
Explain *why* the decision version of this problem is in the class $\mathcal{NP}$.

[10 marks]

(3) (a) State the definition of the **biconnectivity** property in graphs. Explain why the graph given below is not biconnected. Then propose an edge whose addition to the graph will make it biconnected.

[10 marks]



(b) Define the **Fractional Knapsack Problem** and the $\{0, 1\}$ **Knapsack Problem**, explaining carefully how they differ.

What is the name of the general solution method that we can use to solve the Fractional Knapsack Problem, and what is the *different* solution method that we can use to solve the $\{0, 1\}$ Knapsack Problem?

[10 marks]

(4) (a) What is the main observation used in Euclid's algorithm? Trace Euclid's algorithm on the inputs 108 and 63. **[10 marks]**

(b) Clearly state the definition of the following $\mathcal{NP}$-complete problems giving the form that an **instance** of the problem has, as well as the **question** being asked for such an instance.

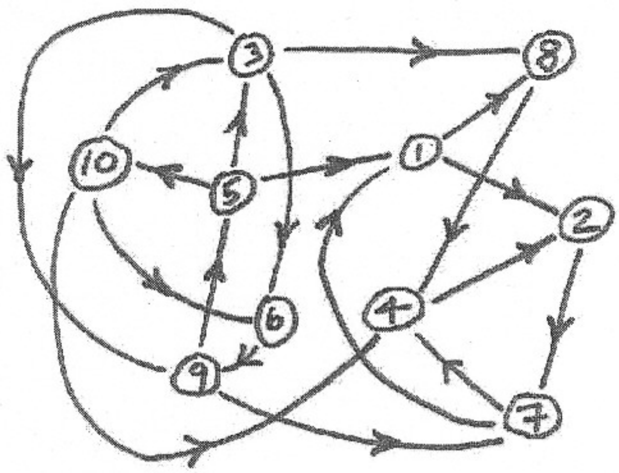
(i) The **Hamilton Cycle Problem** (HC). **[5 marks]**

(ii) The **3-Colouring Problem** (3-COL). **[5 marks]**

(5) (a) Insert the elements of the sequence $S = \{11, 8, 9, 5, 3, 2, 7\}$ into an initially empty heap H. The elements are inserted one at a time in the order of their appearance in H. Draw a tree representation of H after the insertion of each element. Finally give a vector representation of H after the last insertion took place. **[15 marks]**

(b) State necessary and sufficient conditions for a connected undirected graph $G = (V, E)$ to have an Eulerian tour. **[5 marks]**

(6) (a) Consider the directed graph below.



For each set of vertices given below, state whether or not they are in the same strongly connected component, giving a brief justification for your answer.

(i) Vertices 7 and 8. **[3 marks]**

(ii) Vertices 9 and 10. **[3 marks]**

(iii) Vertices 8, 9, and 10. **[4 marks]**

(b) Explain what a *substitution cipher* is. What is a *Caesar cipher*? Comment on how a Caesar cipher can be broken. **[10 marks]**

END OF PAPER