| PAPER CODE NO. COMP108 | EXAMINER : **Dr LA Gasieniec**<br>DEPARTMENT : Computer Science Tel. No. **43686/7910** |
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# SUMMER 2002 EXAMINATIONS

Bachelor of Arts : Year 1
Bachelor of Science : Year 1
Bachelor of Science : Year 2
Master of Mathematics : Year 2

## ALGORITHMIC FOUNDATIONS

**TIME ALLOWED : Two Hours**

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**INSTRUCTIONS TO CANDIDATES**

Answer ***FOUR*** questions

If you attempt to answer more than the required number of questions (in any section), the marks awarded for the excess questions will be discarded (starting with your lowest mark).

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Question 1

**1.A** Prove or disprove using a truth table that:

$$(P => Q) \textbf{ or } R$$

is logically equivalent to:

$$(P => R) \textbf{ or } Q$$

**[10 marks]**

**1.B** Give a contrapositive proof that for any integer $n$ and $m$

$n + m$ *is odd* $=>$ *precisely one of* $n$ *and* $m$ *is even*

is true.

**[10 marks]**

**1.C** State the definition of the *symmetric difference* of two sets $A$ and $B$. Illustrate the definition of the symmetric difference using Venn diagram

**[5 marks]**

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Question 2

**2.A** Prove, using mathematical induction, that

$$1 + 3 + 5 + ... + (2n - 1) = n^2.$$

[10 marks]

**2.B** Professor Einstein is paid every other week on Friday. Show using the pigeonhole principle that in some month he is paid three times.

[10 marks]

**2.C** How many distinct rearrangements are there of the letters in the word *HOCUSPOCUS*.

[5 marks]

Question 3

**3.A** Consider relation $R \subseteq \mathbf{Z} \times \mathbf{Z}$, s.t., for any $x, y \in \mathbf{Z}$, $xRy$ if and only if $x + y$ is an even integer. Prove that $R$ is an *equivalence relation*.

[15 marks]

**3.B** Is the function $f(x) = x+1$ *bijective* when the domain and the codomain are $\mathbf{N}$? Is it bijective when the domain and the codomain are $\mathbf{Z}$?

Justify your answers.

[10 marks]

## Question 4

**4.A** Let $\mathcal{A}_n$ be a nonempty sequence of real numbers $a_1, a_2, a_3, ..., a_n$. A partial sum $S_k$ is a sum of $k$ initial elements in the sequence $\mathcal{A}_n$, i.e., $S_k = a_1+a_2+a_3+ ... + a_k$, for all $k = 1, 2, 3, ..., n$. Design and write pseudocode of a procedure deciding whether all partial sums $S_k$ in sequence $\mathcal{A}_n$ are non-negative ($\geq 0$). What is the complexity of your solution?

**[15 marks]**

**4.B** Trace the values of $i$ and $j$ in the following algorithm when $m = 2$ and $n = 5$.

```
begin
    Input m, n;
    i := 0; j := 1;
    while i < n do
        begin
            i := i + 1; j := j · m;
        end
    Output j;
end
```

What is the output of the algorithm for general $n$ and $m$?

**[10 marks]**

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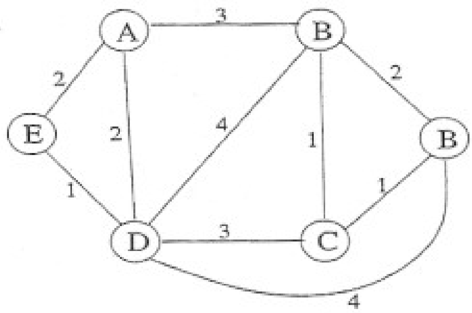
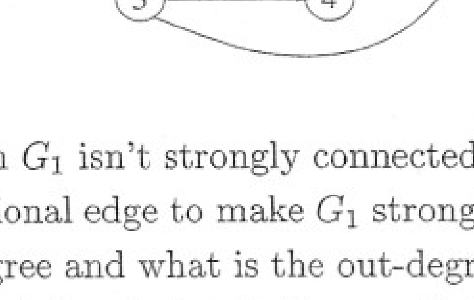
## Question 5

**5.A** The graph $G_2$ represents a road network connecting a set of six towns where distances are given in miles. Use Prim's algorithm to find a road network of minimal total length connecting all the towns:




[15 marks]

**5.B** Consider the following directed graph $G_1$:




1. Explain why graph $G_1$ isn't strongly connected. [2 marks]
2. Propose one additional edge to make $G_1$ strongly connected. [2 marks]
3. What is the in-degree and what is the out-degree of node 5? [2 marks]
4. What is the shortest directed path from node 1 to 3? [2 marks]
5. Is there any cycle in $G_1$ of size 5? [2 marks]