MATH171501

This question paper consists of 5 printed pages, each of which is identified by the reference **MATH171501**.

Statistical tables are attached at the end of the question paper. Only approved basic scientific calculators may be used.

Examination for the Module MATH1715

(January 2005)

INTRODUCTION TO PROBABILITY

Time allowed: **2 hours**

Attempt ALL questions in Section A and THREE questions from Section B.

For Section A only write down a single letter answer for each question.
Section A is worth 40% of the examination marks.

All questions within each section carry equal marks.

Section A

A1. Two radio stations, A and B, broadcast on the same wavelength. In any particular second,

- the probability that A is broadcasting is 0.6;
- the probability that B is broadcasting is 0.8;
- the probability that neither is broadcasting is 0.35.

What is the probability that in any particular second the two stations are jamming each other?

A: 0.48 B: 0.52 C: 0.25 D: 0.92 E: 0.75

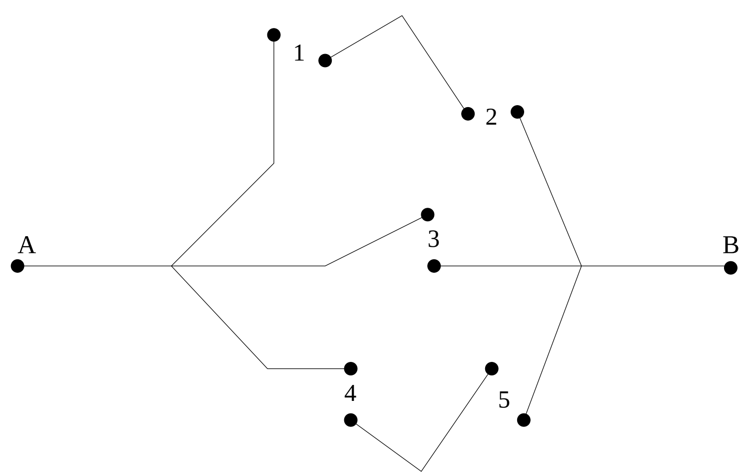
A2. A contractor has 8 suppliers from which to purchase electrical supplies. He will select 3 of these at random and ask each supplier to submit a project bid. If your firm is one of the 8 suppliers, what is the probability that you will get the opportunity to bid on the project?

A: 0.125 B: 0.375 C: 0.250 D: 0.5 E: 0.568

A3. Three factories manufacture 20%, 30% and 50% of computer chips a company sells. If the fractions of defective chips are 0.4%, 0.3% and 0.2%, respectively, what fraction of the defective chips come from the third factory?

A: 0.63 B: 0.23 C: 0.37 D: 0.29 E: 0.44

A4. In the circuit shown below each switch is closed with probability $p = 0.3$, independently of all other switches. ('Closed' means that electric current can pass through the switch.) What is the probability that a flow of current is possible between points A and B?



A: 0.30 B: 0.42 C: 0.48 D: 0.52 E: 0.58

CONTINUED...

A5. The train runs every hour and stays at the station for 6 minutes. You have forgotten the timetable, so you arrive at the station at random with uniform distribution over the one-hour gap between two consecutive departure times. What is the probability that you will wait for the next available train no longer than 10 minutes?

A: $\frac{1}{10}$ B: $\frac{2}{15}$ C: $\frac{1}{6}$ D: $\frac{4}{15}$ E: $\frac{11}{30}$

A6. The probability that a certain type of inoculation takes effect is 0.995. What is the probability that at most 2 out of 400 people given the inoculation will find that it has not taken effect?

A: 0.010 B: 0.323 C: 0.406 D: 0.594 E: 0.677

A7. An insurance company writes a policy to the effect that an amount of $\pounds 1000$ must be paid if a certain event A occurs within a year. The company estimates that A will occur within a year with probability 0.02. What should be the premium c (i.e., the amount in pounds the customer is charged when buying the policy) in order that the company's expected profit be 50% of c?

A: 50 B: 40 C: 70 D: 60 E: 30

A8. For a car travelling at 30 miles per hour (mph), the distance required to brake to a stop is normally distributed with mean of 50 feet and a standard deviation of 8 feet. Suppose you are travelling at 30 mph in a residential area and a car moves abruptly into your path at a distance of 60 feet. If you apply your brakes to stop, what is the probability that you will avoid the collision?

A: 0.8944 B: 0.9053 C: 0.0947 D: 0.1056 E: 0.2113

A9. An urn contains one red ball and five black balls. Players A and B draw one ball at a time alternately, without replacement, with A going first. The player who draws the red ball wins the game. What is the probability that player A wins?

A: $\frac{1}{6}$ B: $\frac{1}{3}$ C: $\frac{1}{2}$ D: $\frac{2}{3}$ E: $\frac{11}{12}$

A10. A football team loses each game with probability 0.4, independently of other games. What is the probability the team will lose two games out of six?

A: 0.544 B: 0.160 C: 0.138 D: 0.311 E: 0.261

Section B

B1. (a) A random variable X has probability density function

$$f_X(x) = \begin{cases} a + bx & \text{if } 0 \leq x \leq 2, \\ 0 & \text{otherwise.} \end{cases}$$

Determine the constants a and b so that $\mathsf{E}(X) = 6/5$.

(b) Let X_1, X_2 and X_3 be independent random variables with zero mean and variance 1. Find the correlation coefficient between $Y_1 = 2X_1 - X_2$ and $Y_2 = X_2 + 2X_3$.

(c) Using the normal approximation with continuity correction, estimate the probability that the number of heads in 400 tosses of a fair coin will be exactly 200.

B2. (a) Given the joint probability mass function $p_{XY}(x, y)$ of discrete random variables X and Y, write down the formulae to compute the marginal distributions of X and Y.

Express the condition for X and Y to be independent in terms of the probability mass functions $p_{XY}(x, y)$, $p_X(x)$, and $p_Y(y)$.

(b) Two hunters shoot at a deer, which is hit by exactly one bullet. If the first hunter hits his target with probability 0.3 and the second with probability 0.6, what is the probability that the second hunter killed the deer?

(c) A random number X is picked according to a uniform distribution on $(0, 1)$. What is the probability that the first digit in the decimal expansion of $\sqrt{X}$ is 7?

B3. (a) Suppose that 10% of a certain brand of jelly beans are red. Use the normal approximation to estimate the probability that in a bag of 400 jelly beans there are at least 45 red ones.

(b) An urn contains 49 balls numbered 1 to 49. In a lottery draw, 7 balls are sampled without replacement. What is the expected value of the sum of the sampled numbers?

(c) Suppose that the waiting time in a queue has an exponential distribution with mean of 20 minutes.

What is the probability that standing in the queue will take more that 20 minutes?

Given that 20 minutes have already been spent in the queue, what is the probability that the overall waiting time will be at least one hour?

Give your answers to 3 significant figures.

B4. (a) A random variable X has probability generating function given by

$$G_X(s) = \frac{s}{2} + \frac{s^3}{6} + \frac{s^6}{3}.$$

Obtain the expected value and the variance of X.

(b) Suppose that we roll a die repeatedly until we see each number at least once. Let X be the number of rolls required. Find $\mathsf{E}(X)$.

(c) Suppose that a random variable X has exponential distribution with parameter $\lambda = 2$. Set $Y = 1 - e^{-2X}$. What is the set of all possible values of Y? Obtain the cumulative distribution function of Y and sketch its graph.

Normal Distribution Function Tables

The first table gives

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}y^2} dy$$

and this corresponds to the shaded area in the figure to the right. $\Phi(x)$ is the probability that a random variable, normally distributed with zero mean and unit variance, will be less than or equal to x. When $x < 0$ use $\Phi(x) = 1 - \Phi(-x)$, as the normal distribution with mean zero is symmetric about zero. For interpolation use the formula

$$\Phi(x) \approx \Phi(x_1) + \frac{x - x_1}{x_2 - x_1} \big(\Phi(x_2) - \Phi(x_1)\big)$$

$$(x_1 < x < x_2)$$

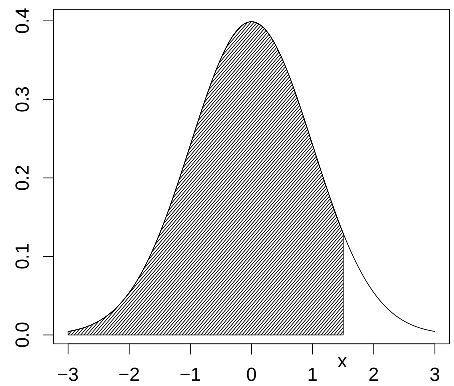


Table 1

x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
0.00	0.5000	**0.50**	0.6915	**1.00**	0.8413	**1.50**	0.9332	**2.00**	0.9772	**2.50**	0.9938
0.05	0.5199	**0.55**	0.7088	**1.05**	0.8531	**1.55**	0.9394	**2.05**	0.9798	**2.55**	0.9946
0.10	0.5398	**0.60**	0.7257	**1.10**	0.8643	**1.60**	0.9452	**2.10**	0.9821	**2.60**	0.9953
0.15	0.5596	**0.65**	0.7422	**1.15**	0.8749	**1.65**	0.9505	**2.15**	0.9842	**2.65**	0.9960
0.20	0.5793	**0.70**	0.7580	**1.20**	0.8849	**1.70**	0.9554	**2.20**	0.9861	**2.70**	0.9965
0.25	0.5987	**0.75**	0.7734	**1.25**	0.8944	**1.75**	0.9599	**2.25**	0.9878	**2.75**	0.9970
0.30	0.6179	**0.80**	0.7881	**1.30**	0.9032	**1.80**	0.9641	**2.30**	0.9893	**2.80**	0.9974
0.35	0.6368	**0.85**	0.8023	**1.35**	0.9115	**1.85**	0.9678	**2.35**	0.9906	**2.85**	0.9978
0.40	0.6554	**0.90**	0.8159	**1.40**	0.9192	**1.90**	0.9713	**2.40**	0.9918	**2.90**	0.9981
0.45	0.6736	**0.95**	0.8289	**1.45**	0.9265	**1.95**	0.9744	**2.45**	0.9929	**2.95**	0.9984
0.50	0.6915	**1.00**	0.8413	**1.50**	0.9332	**2.00**	0.9772	**2.50**	0.9938	**3.00**	0.9987

The inverse function $\Phi^{-1}(p)$ is tabulated below for various values of p.

Table 2

p	**0.900**	**0.950**	**0.975**	**0.990**	**0.995**	**0.999**	**0.9995**
$\Phi^{-1}(p)$	1.2816	1.6449	1.9600	2.3263	2.5758	3.0902	3.2905

END